# Cognition and Behavior in Two-Person Guessing Games Miguel Costa-Gomes, York, and Vincent Crawford, UCSD 

This paper reports experiments that elicit subjects' initial responses to a series of dominance-solvable two-person guessing games, relatives of those studied by Nagel (AER '95) and Ho, Camerer, and Weigelt (AER '98; "HCW")

The design presents the games to subjects with hidden but freely accessible payoff parameters that vary across players and games, to study subjects' cognition via their information search

The goal is to identify more precisely and document more convincingly how subjects' initial responses are determined, and ultimately to develop a structural alternative to Nash equilibrium as a model of initial responses to games

The alternative model should both explain why equilibrium is often a reliable model of initial responses to simple games and predict the systematic deviations that arise in complex games

Such a model would inform many applications that now rely on equilibrium in complex games without clear precedents: e.g. entry, bargaining, auctions, incentives and mechanism design

Accurately modeling initial responses is essential even in applications where people can learn to play an equilibrium, to predict comparative statics or selection among multiple equilibria

It would also help identify the structure of learning, distinguishing reinforcement from beliefs-based and more sophisticated rules

## Previous work on guessing and other games

Nagel's and HCW's guessing or "beauty contest" games were inspired by the famous passage in chapter 12 of Keynes' General Theory, in which he likened professional investment
. . . to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view.

- $n$ (15-18 in Nagel, 3-7 in HCW) subjects guess between limits ( $[0,100]$ in Nagel, $[0,100]$ or $[100,200]$ in HCW)
- The subject whose guess is closest to a target ( $p=1 / 2,2 / 3$, or $4 / 3$ in Nagel, $p=0.7,0.9,1.1$, or 1.3 in HCW) times the group average wins a prize, with ties broken randomly
- The structure is public knowledge
- Subjects played the games repeatedly, but we can view initial responses as "one-shot" if they treated their own influences on future partners' guesses as negligible; plausible in these groups
(These games capture salient features of Keynes' example, but they are not quite the same; compare Van Huyck et al. QJE '91, Mehta et al. AER '94, and Rubinstein, Tversky, and Heller '96)

The [ 0,100 ] games with $p<1$ are dominance-solvable in infinite numbers of rounds, with "all-0" the unique equilibrium (if $p>1$ and $n>2 p$ there are equilibria at all-0 and all-100 but no dominance)

The [100, 200] games are dominance-solvable in finite numbers of rounds, with all-100 the unique equilibrium when $p<1$ and all200 the unique equilibrium when $p>1$

Thus equilibrium predictions of initial responses to these games depend "only" on iterated knowledge of rationality, not of beliefs

Yet Nagel's subjects never played their equilibrium strategies, and HCW's seldom did; instead their initial responses were heterogeneous, most respecting 0 to 3 rounds of dominance

As Keynes concluded,
... It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.

Note that Keynes' wording suggests finite iteration of best responses, initially anchored by players' true preferences

Non-equilibrium responses like those in Nagel's and HCW's experiments are often modeled as "equilibrium plus noise" or "equilibrium taking noise into account" (e.g. McKelvey and Palfrey's GEB '95 quantal response equilibrium or "QRE")

But, even adjusting for the equilibrium being on the boundary, Nagel's and HCW's data resemble neither "noisy Nash" nor QRE for any of the standard distributions

The data do suggest that the deviations from equilibrium have a coherent, non-random, but individually heterogeneous structure

In the $[0,100]$ games, for example, spikes are clearly visible (amid the noise) at $50 p^{k}$ for target $p$ and $k=1,2,3$-like spectrograph peaks that suggest discrete chemical elements

Similar patterns of heterogeneous but structured strategic behavior have been found in initial responses to matrix games by Stahl and Wilson (GEB '95; "SW") and Costa-Gomes et al. (EMT '01; "CGCB"), to alternating-offers bargaining games by Camerer, Johnson et al. ('93, JET '02; "CJ"), and to other games

- Subjects' initial responses are often "strategic" and they make undominated decisions $85-95 \%$ of the time
- Subjects are less likely to rely on dominance for others (Beard and Beil, MS '94), and reliance on iterated dominance stops at 13 rounds (a strategic "constant of nature"?)
- Subjects play equilibrium strategies less often in games where they differ from strategies suggested by certain alternative rules


## Strategic decision rules or "types"

The data from these experiments have been analyzed using certain general decision rules or "types," chosen as plausible descriptions of subjects' behavior, and for theoretical interest

- SW's L1 (CGCB's Naïve) best responds to a uniform prior over its partner's decisions (and so respects one round of dominance); $L 1$ has a perfect model of the game but a naïve (or at least diffuse) model of others' decisions
- CGCB's $L 2$ (L3) best responds to $L 1$ (L2); $L 2(L 3)$ has a perfect model of the game and a less naïve model of others
$L k$ anchors its beliefs with a naïve prior and adjusts them via thought-experiments involving iterated best responses
$L k$ is rational in that it chooses a best response to its beliefs; but those beliefs are based on simplified models of others that don't "close the loop" as equilibrium does

This yields a workable model of others' responses to incentives while avoiding the cognitive complexity of equilibrium analysis

In the words of Selten (EER '98):
"Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties.... Boundedly rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found. Each step of the procedure is simple, even if many case distinctions by simple criteria may have to be made."

Another set of types is closer to how theorists analyze games:

- CGCB's D1 does one round of dominance and then best responds to a uniform prior over its partner's remaining decisions
- D2 does two rounds of dominance and then best responds to a uniform prior over its partner's remaining decisions

Dk starts with iterated knowledge of rationality and then invokes a naïve prior; by standard measures its cognitive requirements are close to $L k+1$ 's, and both respond similarly to dominance

In Nagel's [0, 100] games with $p<1, D k$ 's and $L k+1$ 's guesses are perfectly confounded: $D k$ guesses $\left(\left[0+100 p^{k}\right] / 2\right) p$ and $L k+1$ guesses $[(0+100) / 2] p^{k+1}$; thus both match the spikes

In HCW's [100,200] games and CGCB's matrix games, $D k$ is weakly separated from $L k+1$ and the results are inconclusive

In this paper separating them was an important design goal, and we find that $D k$ subjects are far less frequent than $L k+1$ subjects

Two other types are important in our analysis

- Equilibrium makes equilibrium decisions
- Sophisticated best responds to the probability distributions of others' decisions (estimated from the observed frequencies), the behavioral game theory ideal, included to learn if any subjects have an understanding of others that transcends mechanical rules (we find little evidence of this, but some of Equilibrium)


## New guessing design

In our Baseline treatment, game-theoretically naïve subjects played a series of 16 different two-person guessing games; subjects were anonymously, randomly paired with no feedback during play to suppress learning and repeated-game effects (Our design builds on SW and CGCB in eliciting initial responses to a series of games, and on CJ and CGCB in presenting the games with hidden, freely accessible payoff parameters to study cognition by monitoring subjects' information searches; but it is the first to do either of these things with guessing games)

- In each game, two players make simultaneous guesses
- Each player has a lower and an upper limit, both positive (so with finite dominance-solvability as in HCW's [100, 200] games)
- But players are not required to guess between their limits: instead guesses outside the limits are automatically adjusted up to the lower limit or down to the upper limit as needed
(Payoffs are quasiconcave, so a subject can enter his ideal guess, ignoring his limits, and know without checking his limits that his adjusted guess will be optimal; this separates types' search implications, particularly L1's, more than in other designs)
- Each player has a target, and his payoff increases with the closeness of his adjusted guess to his target times the other's adjusted guess (thus players' guesses determine continuous payoffs rather than who wins an all-or-nothing prize, and payoffs depend on a partner's rather than the group average guess)
- The targets and limits vary independently across players and games, with the targets both less than one, both greater than one, or mixed (in previous guessing experiments the targets and limits were always the same for both players within a treatment)
- Because the targets and limits vary, subjects don't know them
- We use a MouseLab interface to present the games with targets and limits hidden, giving subjects free access to them game by game, publicly announcing all other aspects of the structure (including the fact that subjects have free access)
- Low search costs then make the games' structures effectively public knowledge, so that (with the suppression of learning and repeated-game effects) our design induces a series of 16 independent complete-information games


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The games have a balanced mix of targets, limits, and strategic structures, dominance-solvable in from 2-52 rounds (there are eight player-symmetric pairs and one pair of symmetric games)

Table II. Strategic Structures

| $\begin{gathered} \hline \underset{i}{\text { Game }} \end{gathered}$ | Order Played | Fargets | Equilibrium | Rounds of Dominance | Pattern of Dominance | Dominance at Both ends |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 人2ß1 | 6 | Low | Low | 4 | A | No |
| $\beta 1 \alpha 2$ | 15 | Low | Low | 3 | A | No |
| ß1y2 | 14 | Low | Low | 3 | A | Yes |
| v2ß1 | 10 | Low | Low | 2 | A | No |
| v483 | 9 | High | High | 2 | S | No |
| ठ3v4 | 2 | High | High | 3 | S | Yes |
| ס3ठ3 | 12 | High | High | 5 | S | No |
| ס383 | 3 | High | High | 5 | S | No |
| $\beta 1 \times 4$ | 16 | Mixed | Low | 9 | S/A | No |
| a4ß1 | 11 | Mixed | Low | 10 | S/A | No |
| ס2ß3 | 4 | Mixed | Low | 17 | S/A | No |
| ß382 | 13 | Mixed | Low | 18 | S/A | No |
| v2ß4 | 8 | Mixed | High | 22 | A | No |
| $\beta 4 \mathrm{y} 2$ | 1 | Mixed | High | 23 | A | Yes |
| a2a4 | 7 | Mixed | High | 52 | S/A | No |
| Q4a2 | 5 | Mixed | High | 51 | S/A | No |

Limits: ( $\alpha$ ) 100,500; ( $\beta$ ) 100,900; (ү) 300,500; ס) 300,900
Targets: (1) 0.5; (2) 0.7 ; (3) 1.3; (4) 1.5
Pattern of Dominance: A $\equiv$ Alternating; $\mathrm{S} \equiv$ Simultaneous;
S/A $\equiv$ Alternating in first round, then Simultaneous
The games have essentially unique equilibria determined (not always directly) by players' lower (upper) limits when the product of targets is less (greater) than one ("essentially" only because guesses that lead to the same adjusted guess are equivalent)
E.g. game $ү 2 \beta 4$ : Targets are 0.7 and 1.5 , product is $1.05>1$ so the equilibrium is High; in it the Y 2 player guesses his upper limit 500 , but the $\beta 4$ player guesses 750 , below his upper limit 900

The discontinuity of the equilibrium correspondence when the product of targets equals one enhances separation of equilibrium from boundedly rational rules; games like $\delta 2 \beta 3$ and $ү 2 \beta 4$ differ mainly in whether the product is slightly below or above one; equilibrium responds much more strongly to this than other rules

## Open Boxes and Robot/Trained Subjects Treatments

The Open Boxes ("OB") treatment was identical to the Baseline, but with targets and limits continually visible; we find insignificant differences between Baseline and OB subjects' guesses, suggesting that decisions are not distorted by looking up payoffs

There were six Robot/Trained Subjects ("R/TS") treatments, each identical to the Baseline except that an R/TS subject was trained to identify the guesses implied by a type ( $L 1, L 2, L 3, D 1$, D2, or Equilibrium) and told that he was playing with a robot (framed as "the computer"), which would choose its guesses in the way that justified his assigned type's beliefs

R/TS results provide a benchmark by which to judge the model of cognition and search we use to analyze Baseline results

The R/TS results show that with training, most (though not all) subjects are capable of identifying the types' guesses, so if Baseline subjects' don't make equilibrium guesses, it cannot be attributed entirely to the cognitive difficulty of identifying equilibria

The R/TS results also suggest that $L k$ types are much easier than Equilibrium, which may be easier in turn than $D k$ types

## Advantages of the design

- Tracking behavior within subjects across 16 games with large strategy spaces and varying payoff parameters greatly enhances separation of equilibrium and alternative rules' guesses, yielding sharper identification; e.g. L2 and D1 are much better separated

Types' guesses in the 16 games, in (randomized) order played

|  | L1 | L2 | L3 | D1 | D2 | Eq. | Sop. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 600 | 525 | 630 | 600 | 611.25 | 750 | 630 |
| $\mathbf{2}$ | 520 | 650 | 650 | 617.5 | 650 | 650 | 650 |
| $\mathbf{3}$ | 780 | 900 | 900 | 838.5 | 900 | 900 | 900 |
| $\mathbf{4}$ | 350 | 546 | 318.5 | 451.5 | 423.15 | 300 | 420 |
| $\mathbf{5}$ | 450 | 315 | 472.5 | 337.5 | 341.25 | 500 | 375 |
| $\mathbf{6}$ | 350 | 105 | 122.5 | 122.5 | 122.5 | 100 | 122 |
| $\mathbf{7}$ | 210 | 315 | 220.5 | 227.5 | 227.5 | 350 | 262 |
| $\mathbf{8}$ | 350 | 420 | 367.5 | 420 | 420 | 500 | 420 |
| $\mathbf{9}$ | 500 | 500 | 500 | 500 | 500 | 500 | 500 |
| $\mathbf{1 0}$ | 350 | 300 | 300 | 300 | 300 | 300 | 300 |
| $\mathbf{1 1}$ | 500 | 225 | 375 | 262.5 | 262.5 | 150 | 300 |
| $\mathbf{1 2}$ | 780 | 900 | 900 | 838.5 | 900 | 900 | 900 |
| $\mathbf{1 3}$ | 780 | 455 | 709.8 | 604.5 | 604.5 | 390 | 695 |
| $\mathbf{1 4}$ | 200 | 175 | 150 | 200 | 150 | 150 | 162 |
| $\mathbf{1 5}$ | 150 | 175 | 100 | 150 | 100 | 100 | 132 |
| $\mathbf{1 6}$ | 150 | 250 | 112.5 | 162.5 | 131.25 | 100 | 187 |

Table IV. Numbers of games in which types' guesses are separated*

|  | L1 | L2 | L3 | $\boldsymbol{D 1}$ | $\boldsymbol{D 2}$ | Eq. | Sop. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{L 1}$ | - | 15,13 | 15,12 | 12,10 | 15,12 | 15,15 | 15,14 |
| L2 | 15,13 | - | 11,9 | 13,9 | 10,8 | 11,9 | 10,8 |
| L3 | 15,12 | 11,9 | - | 13,12 | 8,5 | 9,6 | 9,8 |
| $\boldsymbol{D 1}$ | 12,10 | 13,9 | 13,12 | - | 9,7 | 14,13 | 12,10 |
| $\boldsymbol{D 2}$ | 15,12 | 10,8 | 8,5 | 9,7 | - | 9,8 | 9,6 |
| Eq. | 15,15 | 11,9 | 9,6 | 14,13 | 9,8 | - | 11,9 |
| Sop. | 15,14 | 10,8 | 9,8 | 12,10 | 9,6 | 11,9 | - |

By more than 0 (or 0.5 ), by more than 25

- Two-person guessing games focus sharply on the central strategic problem of predicting the decisions of others who view themselves as a non-negligible part of one's own environment
- Although our games are not zero-sum and have more than two possible payoffs, like other guessing games they limit the effects of altruism, spite, and (by design of payoff function) risk aversion
- Varying the targets and limits within a common structure greatly enhances separation of types' search implications and makes monitoring search a powerful tool for studying cognition
- It also makes each type's search implications independent of the game (with one minor exception), which often allows us to read a subject's type directly from his information search pattern
- As in other guessing games, the intuitive common structure reduces the noisiness typical of initial responses to games
- It also makes mental models of others easy to express as functions of the targets and limits, which seems to encourage subjects to articulate such models to themselves; this enhances the clarity of the results, but might also distort subjects' guesses


## Studying cognition via guesses and information search

"The look-ups are the windows of the strategic soul." -folk saying of the MouseLab people

We link guesses and search by assuming each subject has a single, pure type, which determines them in the 16 games

The types L1, L2, L3, D1, D2, Equilibrium, and Sophisticated were chosen for appropriateness as possible descriptions of behavior, from general principles that have played important roles in the literature (CGCB's Altruistic, Optimistic, and Pessimistic have limited relevance in these games)

A priori specification seems necessary because a type's search implications depend not only on what it guesses, but why

These types provide a kind of basis for the enormous space of possible guesses and searches, imposing enough structure to make it meaningful to ask if they are related in a coherent way

Table VI summarizes our characterization of types' ideal guesses (which determine their adjusted guesses via the automatic adjustment function $R())$ and search implications

Table VI: Types' Ideal Guesses and Relevant Look-ups

| Type | Ideal guess | Search implications |
| :---: | :---: | :---: |
| L1 | $p^{\prime}\left[a^{\prime}+b^{\prime}\right] / 2$ | $\left\{\left[a^{\prime}, b^{\prime}\right], p^{\prime}\right\} \equiv\{[4,6], 2\}$ |
| L2 | $p^{i} R\left(a^{j}, b^{\prime} ; p^{j}\left[a^{i}+b^{j}\right] / 2\right)$ | $\left\{\left(\left[a^{i}, b^{\prime}\right], p^{j}\right), a^{j}, b^{j}, p^{\prime}\right\} \equiv\{([1,3], 5), 4,6,2\}$ |
| L3 | $p^{i} R\left(a^{j}, b^{j} ; p^{j} R\left(a^{i}, b^{i} ; p^{\prime}\left[a^{j}+b^{j}\right] / 2\right)\right)$ | $\left\{\left(\left[a^{j}, b^{\prime}\right], p^{\prime}\right), a^{i}, b^{i}, p^{\prime}\right\} \equiv\{([4,6], 2), 1,3,5\}$ |
| D1 | $p^{\prime}\left(\max \left\{a^{j}, p^{j} a^{i}\right\}+\min \left\{p^{i} b^{i}, b^{\prime}\right\}\right) / 2$ | $\left\{\left(a^{j},\left[p^{j}, a^{\prime}\right]\right),\left(b^{j},\left[p^{j}, b^{\prime}\right]\right), p^{\prime}\right\} \equiv\{(4,[5,1]),(6,[5,3]), 2\}$ |
| D2 | $p^{j}\left\{\max \left\{\max \left\{a^{j}, p^{j} a^{i}\right\}, p^{j} \max \left\{a^{i}, p^{i} a^{j}\right\}\right\}\right.$ $\left.+\min \left\{p^{j} \min \left\{p^{i} b^{j}, b^{\prime}\right\}, \min \left\{j^{j} b^{i}, b^{j}\right\}\right\}\right] / 2$ | $\begin{aligned} & \left\{\left(a^{i},\left[p^{i}, j^{a}\right]\right),\left(b^{i},\left[p^{i}, b^{j}\right]\right),\left(a^{j},\left[p^{j}, a^{j}\right]\right),\left(b^{j},\left[p^{j}, b^{j}\right]\right), p^{j}, p^{i}\right\} \\ & =\{(1,[2,4]),(3,[2,6]),(4,[5,1]),(6,[5,3]), 5,2\} \end{aligned}$ |
| Eq. | $p^{i} a^{j}$ if $p^{i} p^{j}<1$ or $p^{i} b^{j}$ if $p^{i} p^{j}>1$ | $\begin{gathered} \left\{\left[p^{i}, p^{j}\right], a^{i}\right\} \equiv\{[2,5], 4\} \text { if } p^{i} p^{i}<1 \\ \text { or }\left\{\left[p^{i}, p^{j}\right], b^{\prime}\right\} \equiv\{[2,5], 6\} \text { if } p^{i} p^{\prime}>1 \end{gathered}$ |
| Sop. | [no closed-form expression; search implications are the same as D2's] | $\begin{aligned} & \left\{\left(a^{i},\left[p^{i}, a^{a}\right]\right),\left(b^{i},\left[p^{j}, b^{j}\right]\right),\left(a^{j},\left[p^{j}, a^{j}\right]\right),\left(b^{j},\left[p^{j}, b^{j}\right]\right), p^{j}, p^{i}\right\} \\ & \stackrel{\equiv\{(1,[2,4]),(3,[2,6]),(4,[5,1]),(6,[5,3]), 5,2\}}{ } \end{aligned}$ |

$p$ is a target; $a(b)$ is a lower (upper) limit; $i$ and $j$ are the player and his partner; and $R()$ is the automatic adjustment function. Basic operations-the innermost operations in the formulas on the left side, in square brackets on the right-must be associated with adjacent look-ups, which can appear in any order but can't be separated; other operations, in parentheses or curly brackets on the right side, can appear in any order, and can be separated. The table gives the order that seems most natural when there is one, but we do not insist on this order.

## Derivation of types' ideal guesses

Subjects are paid for their points in $5 / 16$ randomly selected games, so if a subject maximizes the expected utility of his money payment, and guesses have known consequences, he will maximize his point payoff in any given game

Because equilibrium always implies unique, known adjusted guesses, risk preferences do not affect Equilibrium guesses

Our payoff function makes best responses to uniform priors on intervals certainty-equivalent (due to symmetry of the function), so risk preferences do not affect $L k$ or $D k$ guesses either

But risk preferences might affect Sophisticated guesses, and for it we assume risk-neutrality for simplicity

Each of our types implies a unique, pure ideal guess in each game (Sophisticated only generically)

Certainty-equivalence and our characterization of equilibrium immediately yield types' ideal guesses, except Sophisticated's, which we compute from the data

## Derivation of types' search implications

Standard assumptions imply that a type will look up all freely available information that might affect its guess

Each type is naturally associated with algorithms that describe how to process this information into a guess

We use a type's algorithms as models of cognition, and derive the search implications of those algorithms under conservative assumptions about how cognition affects search (Assumptions are needed because if a subject memorized parameters, look-up order could be unrelated to cognition; our assumptions are corroborated by our R/TS treatments)

Because a subject can enter his ideal guess and know that his adjusted guess will be optimal, and we seek minimal search implications, we derive them from ideal guesses

We assume that basic operations are associated with adjacent look-ups, which can appear in any order but cannot be separated; other operations can appear in any order, and can be separated; the table gives the order that seems most natural when there is one, but we do not insist on this order

In the econometric analysis we summarize a subject's compliance with a type's search implications in a game, under these assumptions, by the density of the type's look-up sequence in the subject's observed look-up sequence

L1, L2, L3, D1, D2 search implications are easy to derive
Equilibrium can use any workable method to find its ideal guess, which equals its target times its partner's lower (upper) limit when the product of targets is < (>) 1

In particular it can conjecture and check guesses for consistency with equilibrium, which is less demanding than other methods and determines minimal search implications, but requires more luck than our subjects appeared to have

Using our characterization of equilibrium yields the same look-up requirements but requires targets to be adjacent; we take this to determine Equilibrium's search implications

Note that unlike in CGCB's design, Equilibrium's minimal search implications are as simple as L1's, and simpler than other boundedly rational types'

We assume that Sophisticated, like a good behavioral game theorist, must deduce its beliefs from the game's structure, including its equilibrium and dominance relationships

But we ignore more than two rounds of dominance as impractical, and take Sophisticated's search implications as the union of Equilibrium's and D2's; because D2's include Equilibrium's, Sophisticated's implications reduce to D2's

## Results for Baseline and OB subjects' guesses

On average 90\% of Baseline and OB subjects' guesses respected simple dominance, much more than random ( $\sim 60 \%$ here) and typical of initial responses to games

All but 12 respected dominance in 13 or more games (80\%), suggesting that they understood the games and maximized self-interested expected payoffs, given coherent beliefs

43 of 88 subjects made $7-16$ of some type's exact (within 0.5 ) guesses: far more than could occur by chance, given the strong separation of types' guesses and the fact that guesses could take from 200 to 800 different rounded values

But 35 of those 43 subjects conformed closely to types other than Equilibrium: 20 to L1, 12 to L2, and 3 to L3

Given our type definitions, those subjects' deviations from equilibrium can be confidently ascribed to non-equilibrium beliefs rather than altruism, spite, confusion, or irrationality

The results for guesses also favor CGCB's noiseless definition of $L k, k>1$, over SW's, which best responds to a noisy Lk-1; and provide evidence against types that depend on estimated population parameters, such as SW's Worldly

Table IX gives subject by subject maximum likelihood type estimates based on guesses for a spike-logit error structure, in which a subject of a given type has probability $1-\varepsilon$ of making his type's guess exactly (within 0.5 ) in a game, and his guess otherwise has a logit distribution with precision $\lambda$

Point type estimates assign 43 subjects to $L 1,20$ to $L 2,3$ to L3, 5 to D1, 14 to Equilibrium, and 3 to Sophisticated

Subjects make exact guesses so often that the spike is needed for 81 (83) of our 88 subjects at the $1 \%(5 \%)$ level

Spike-logit does significantly better than a spike-uniform error structure for only 21 (34) subjects at the 1\% (5\%) level

All but 10 subjects' estimated types do significantly better than a completely random model of guesses; the estimated types that fail this test are marked " $\dagger$ " in the table

Table IX. Type Estimates Based on Guesses Only, Search Only, and Guesses and Search

|  |  | Guesses | only |  |  | Search | only |  |  | Guesses | and | search |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ID | dom. | $\ln \mathrm{L}$ | $k$ | exact | $\lambda$ | $\ln \mathrm{L}$ | $\boldsymbol{k}_{s}$ | $\zeta_{H}$ | $\zeta_{M}$ | $\boldsymbol{l n} \mathrm{L}_{\mathrm{t}}$ | $\boldsymbol{n} \mathrm{L}_{\mathrm{g}}$ | $\boldsymbol{l n} \mathrm{L}_{\mathrm{s}}$ | $k_{s}$ | exact | $\lambda$ | $\zeta_{H}$ | $\zeta_{M}$ |
| 513 | 0 | 0.00 | L1 | 16 | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 118 | 0 | -9.62 | L1 | 15 | 1.85 | -7.41 | $L 1_{e}$ | 0.88 | 0.06 | -17.03 | -9.62 | -7.41 | $L 1_{e}$ | 15 | 1.85 | 0.88 | 0.06 |
| 101 | 1 | -10.27 | L1 | 15 | 0.55 | -9.94 | $L 1_{e}{ }^{\ddagger}$ | 0.69 | 0.31 | -20.21 | -10.27 | -9.94 | $L 1_{e}{ }^{\text {\# }}$ | 15 | 0.55 | 0.69 | 0.31 |
| 104 | 0 | -16.63 | L1 | 14 | 2.20 * | -3.74 | $L 1_{e}$ | 0.00 | 0.94 | -20.37 | -16.63 | -3.74 | $L 1_{e}$ | 14 | 2.20 | 0.00 | 0.94 |
| 413 | 0 | -17.81 | L1 | 14 | 0.88 | -6.03 | L1 ${ }_{l}$ | 0.13 | 0.88 | -23.84 | -17.81 | -6.03 | $L 1_{l}$ | 14 | 0.88 | 0.13 | 0.88 |
| 207 | 0 | -17.96 | L1 | 14 | 0.42 | 0.00 | $L 1_{e}$ | 1.00 | 0.00 | -17.96 | -17.96 | 0.00 | $L 1_{e}$ | 14 | 0.42 | 1.00 | 0.00 |
| 216 | 1 | -25.41 | L1 | 13 | 1.06 | -11.25 | $L 3_{e}$ | 0.75 | 0.19 | -38.69 | -25.41 | -13.29 | $L 1_{e}$ | 13 | 1.06 | 0.31 | 0.63 |
| 402 | 0 | -30.93 | L1 | 12 | 5.65* | -9.00 | $L 1_{e}$ | 0.00 | 0.75 | -39.93 | -30.93 | -9.00 | $L 1_{e}$ | 12 | 5.65 | 0.00 | 0.75 |
| 418 | 0 | -42.23 | L1 | 10 | $21.22^{* *}$ | -7.41 | $L 2_{e}$ | 0.88 | 0.06 | -52.16 | -42.23 | -9.94 | $L 1_{e}$ | 10 | 21.22 | 0.00 | 0.69 |
| 301 | 1 | -45.84 | $L 1^{\text {D }}$ | 10 | 0.00 | -3.74 | $L 1_{e}$ | 0.06 | 0.94 | -49.58 | -45.84 | -3.74 | $L 1_{e}$ | 10 | 0.00 | 0.06 | 0.94 |
| 508 | 0 | -46.19 | $L 1^{\text {D }}$ | 10 | 2.05 | - | e | - | - | - | - | - | e | - | - | - | - |
| 308 | 3 | -47.34 | L1 | 10 | 0.00 | -9.63 | $L 3_{e}$ | 0.81 | 0.13 | -60.65 | -47.34 | -13.30 | $L 1_{e l}$ | 10 | 0.00 | 0.19 | 0.69 |
| 102 | 4 | -47.63 | L1 | 10 | 0.00 | -9.63 | $L 2_{e}$ | 0.81 | 0.06 | -57.57 | -47.63 | -9.94 | L1 ${ }_{e}$ | 10 | 0.00 | 0.00 | 0.69 |
| 415 | 1 | -53.64 | L1 | 9 | 0.88 | -16.38 | $D 1_{e}$ | 0.31 | 0.50 | -107.28 | -90.90 | -16.38 | $D 1_{e}$ | 2 | 0.76 | 0.31 | 0.50 |
| 504 | 1 | -56.97 | L1 | 8 | $1.68{ }^{* *}$ | - | , | - | - | - | - | - | - | - | - | - | - |
| 208 | 6 | -61.62 | L1 | 8 | 0.00 | -3.74 | L1 ${ }_{l}$ | 0.06 | 0.94 | -65.37 | -61.62 | -3.74 | L1 ${ }_{l}$ | 8 | 0.00 | 0.06 | 0.94 |
| 318 | 0 | -62.61 | L1 | 7 | $3.18{ }^{*}$ | -3.74 | $L 1_{e}^{\ddagger}$ | 0.00 | 0.94 | -66.36 | -62.61 | -3.74 | $L 1_{e}$ | 7 | 3.18 | 0.00 | 0.94 |
| 512 | 0 | -63.33 | L1 | 7 | 1.56 | - | - | - | - | - | - | - | - | - | - | - | - |
| 502 | 1 | -64.55 | L1 | 7 | 1.01 | - | - | - | - | - | - | - | - | - | - | - | - |
| 516 | 1 | -64.93 | L1 ${ }^{\text {C }}$ | 7 | $1.10{ }^{*}$ | - | - | - | - | - | - | - | - | - | - | - | - |
| 409 | 0 | -73.59 | $L 1^{\text {E }}$ | 4 | 9.90** | -10.59 | L1 ${ }_{l}$ | 0.00 | 0.38 | -84.18 | -73.59 | -10.59 | L1 ${ }_{1}$ | 4 | 9.90 | 0.00 | 0.38 |
| 106 | 0 | -75.82 | L1 | 5 | 1.19 | -7.72 | $E q_{e}$ | 0.00 | 0.19 | -85.75 | -75.82 | -9.94 | $L 1_{l}$ | 5 | 1.19 | 0.00 | 0.31 |
| 305 | 3 | -79.89 | L1 | 5 | 0.37 | -6.03 | $L 1_{e}$ | 0.88 | 0.13 | -85.92 | -79.89 | -6.03 | $L 1_{e}$ | 5 | 0.37 | 0.88 | 0.13 |
| 411 | 1 | -80.58 | L1 | 4 | $1.45{ }^{* *}$ | 0.00 | $L 3_{e}$ | 1.00 | 0.00 | -86.61 | -80.58 | -6.03 | $L 1_{e}$ | 4 | 1.45 | 0.13 | 0.88 |
| 509 | 1 | -81.81 | L1 | 4 | 0.86 | - |  | - | - | - | - | - | - | - | - | - | - |
| 203 | 4 | -83.90 | L1 | 4 | 0.00 | -9.94 | $E q_{e}$ | 0.00 | 0.31 | -94.49 | -83.90 | -10.59 | $L 1_{e}$ | 4 | 0.00 | 0.00 | 0.63 |
| 505 | 4 | -84.13 | L1 | 4 | 0.43 | - |  | - | - | - | - | - | - | - | - | - | - |
| 317 | 3 | -86.58 | L1 | 3 |  |  |  |  | 0.06 | -90.32 | -86.58 | -3.74 | $L 1_{e}$ | 3 | 0.92 | 0.94 | 0.06 |
| 416 | 1 | -86.74 | $L 1^{\dagger}$ | 1 | $4.48{ }^{* *}$ | -3.74 | $L 1_{e}{ }^{\ddagger}$ | 0.00 | 0.94 | -90.48 | -86.74 | -3.74 | $L 1_{e}$ | 1 | 4.48 | 0.00 | 0.94 |


| 217 | 3 | -87.12 | L1 | 3 | 0.68 | -10.59 | $L 1_{e}$ | 0.00 | 0.38 | -97.71 | -87.12 | -10.59 | L1e | 3 | 0.68 | 0.00 | 0.38 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 219 | 3 | -87.32 | L1 ${ }^{+}$ | 3 | 0.89* | -7.72 | $L 1_{e}$ | 0.00 | 0.81 | -95.04 | -87.32 | -7.72 | L1e | 3 | 0.89 | 0.00 | 0.81 |
| 501 | 1 | -87.93 | $L 1^{\dagger}$ | 0 | 4.38** | - | - | - | - | - | - | - | - | - |  | - | - |
| 410 | 3 | -89.18 | L1 | 2 | $1.53 * *$ | -7.72 | $L 1_{e l}{ }^{\ddagger}$ | 0.00 | 0.19 | -96.90 | -89.18 | -7.72 | $L 1_{e l}$ | 2 | 1.53 | 0.00 | 0.19 |
| 510 | 5 | -89.60 | L1 | 3 | 0.00 | - | - | - | - | - | - | - | - | - | - | - | - |
| 420 | 2 | -89.68 | L1 ${ }^{+}$ | 2 | $1.25{ }^{* *}$ | -3.74 | $E q_{l}$ | 0.00 | 0.06 | -94.26 | -90.52 | -3.74 | $E q_{l}$ | 3 | 0.19 | 0.00 | 0.06 |
| 408 | 2 | -89.71 | L1 ${ }^{+}$ | 2 | 1.09** | -6.03 | L1e | 0.00 | 0.88 | -95.74 | -89.71 | -6.03 | L1e | 2 | 1.09 | 0.00 | 0.88 |
| 201 | 3 | -90.26 | L1 ${ }^{+}$ | 2 | $1.21{ }^{* *}$ | -3.74 | $L 1_{e}{ }^{\ddagger}$ | 0.00 | 0.94 | -94.00 | -90.26 | -3.74 | $L 1_{e}$ | 2 | 1.21 | 0.00 | 0.94 |
| 105 | 2 | -90.58 | $\mathrm{L1}^{+}$ | 2 | $1.29 * *$ | -9.00 | $E q_{e}$ | 0.25 | 0.75 | -102.56 | -93.56 | -9.00 | $E q_{e}$ | 2 | 0.11 | 0.25 | 0.75 |
| 103 | 3 | -90.61 | L1 ${ }^{+}$ | 2 | 1.12* | -6.03 | $L 1_{e}$ | 0.00 | 0.13 | -96.63 | -90.61 | -6.03 | $L 1_{e}$ | 2 | 1.12 | 0.00 | 0.13 |
| 213 | 2 | -95.57 | $L 1^{\text {++ }}$ | 0 | 1.19* | -3.74 | $L 2_{e}$ | 0.94 | 0.00 | -100.34 | -96.60 | -3.74 | $L 2_{e}$ | 0 | 0.62 | 0.94 | 0.00 |
| 515 | 4 | -95.68 | $L 1^{\text {+ }}$ | 1 | 0.60 | - | 硡 | - | - | - | - | - | - | - | - | - | - |
| 113 | 5 | -96.61 | $L 1^{\text {++ }}$ | 1 | 0.07 | -9.63 | $L 3_{e l}{ }^{\ddagger}$ | 0.81 | 0.06 | -108.49 | -98.86 | -9.63 | $L 3_{e l}$ | 4 | 0 | 0.81 | 0.06 |
| 109 | 8 | -97.31 | $L 1^{\dagger+}$ | 1 | 0.00 | - | - | - | - | - | - | - | - | - | - | - | - |
| 309 | 0 | 0.00 | L2 | 16 | - | -9.94 | $L 2{ }_{e l}{ }^{\ddagger}$ | 0.69 | 0.00 | -9.94 | 0.00 | -9.94 | $L 2_{e l}$ | 16 | 0.00 | 0.69 | 0.00 |
| 405 | 0 | 0.00 | L2 | 16 | - | -13.30 | $L 3_{e}$ | 0.69 | 0.13 | -14.40 | 0.00 | -14.40 | L2e | 16 | 0.00 | 0.63 | 0.25 |
| 206 | 0 | -10.07 | L2 | 15 | 0.79 | -7.41 | $L 2_{e}$ | 0.88 | 0.06 | -17.49 | -10.07 | -7.41 | L2e | 15 | 0.79 | 0.88 | 0.06 |
| 209 | 0 | -25.51 | L2 | 13 | 0.96 | -9.00 | $L 1_{e}$ | 0.00 | 0.75 | -35.45 | -25.51 | -9.94 | L2 ${ }_{1}$ | 13 | 0.96 | 0.69 | 0.31 |
| 108 | 0 | -25.88 | L2 | 13 | 0.45* | 0.00 | $L 2_{e}{ }^{\ddagger}$ | 1.00 | 0.00 | -25.88 | -25.88 | 0.00 | $L 2_{e}$ | 13 | 0.45 | 1.00 | 0.00 |
| 214 | 2 | -35.30 | L2 | 11 | 2.73 ** | -3.74 | $L 1_{e}$ | 0.00 | 0.94 | -41.33 | -35.30 | -6.03 | L2e | 11 | 2.73 | 0.88 | 0.13 |
| 307 | 1 | -38.88 | L2 | 11 | 1.04* | -7.72 | $E q_{e}$ | 0.00 | 0.19 | -48.51 | -38.88 | -9.63 | L2 ${ }_{1}$ | 11 | 1.04 | 0.81 | 0.13 |
| 218 | 0 | -40.54 | L2 | 11 | 0.60 | -7.72 | $L 1_{e}$ | 0.00 | 0.81 | -53.84 | -40.54 | -13.30 | L2 ${ }_{1}$ | 11 | 0.60 | 0.69 | 0.19 |
| 422 | 2 | -55.79 | L2 | 9 | 0.22 | 0.00 | $L 1_{e}$ | 0.00 | 1.00 | -61.82 | -55.79 | -6.03 | L2e | 9 | 0.22 | 0.88 | 0.13 |
| 316 | 1 | -58.43 | L2 | 8 | 0.73 | -10.97 | $E q_{e}{ }^{\ddagger}$ | 0.00 | 0.44 | -72.26 | -58.43 | -13.84 | L2 ${ }_{1}$ | 8 | 0.73 | 0.06 | 0.38 |
| 407 | 0 | -60.98 | L2 ${ }^{\text {C }}$ | 8 | 0.44 | -6.03 | $L 2_{e}{ }^{\ddagger}$ | 0.88 | 0.13 | -67.00 | -60.98 | -6.03 | $L 2_{e}$ | 8 | 0.44 | 0.88 | 0.13 |
| 306 | 2 | -68.48 | L2 | 7 | 0.18 | -3.74 | L1 | 0.00 | 0.06 | -75.68 | -71.94 | -3.74 | L1 ${ }_{1}$ | 6 | 0.71 | 0.00 | 0.06 |
| 412 | 0 | -69.43 | L2 | 6 | $1.05{ }^{* *}$ | 0.00 | $L 2_{e}{ }^{\ddagger}$ | 1.00 | 0.00 | -69.43 | -69.43 | 0.00 | L2e | 6 | 1.05 | 1.00 | 0.00 |
| 205 | 0 | -72.81 | L2 | 6 | 0.01 | 0.00 | $L 1_{e}$ | 0.00 | 1.00 | -75.80 | -75.80 | 0.00 | L1e | 4 | 3.27 | 0.00 | 1.00 |
| 220 | 1 | -72.96 | L2 | 6 | 0.32 | 0.00 | L1e | 0.00 | 1.00 | -76.70 | -72.96 | -3.74 | $L 2_{e}$ | 6 | 0.32 | 0.94 | 0.06 |
| 403 | 0 | -73.60 | L2 | 6 | 0.50 | -6.03 | $E q_{l}{ }^{\ddagger}$ | 0.00 | 0.13 | -86.91 | -80.88 | -6.03 | $E q_{l}$ | 4 | 0.84 | 0.00 | 0.13 |
| 517 | 0 | -73.70 | L2 | 5 | $0.98{ }^{* *}$ | - | - | - | - | - | - | - | - | - | - | - | - |
| 503 | 3 | -88.21 | $\mathrm{L2}^{+}$ | 3 | 0.00 | - | - | - | - | - | - | - | - | - | - | - | - |


| 414 | 4 | -89.00 | L2 | 2 | 0.78* | -7.72 | $L 1_{e}$ | 0.00 | 0.19 | -102.56 | -92.62 | -9.94 | $E q_{e}$ | 2 | 0.36 | 0.00 | 0.31 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 110 | 3 | -92.51 | L2 ${ }^{+}$ | 2 | 0.00 | -9.00 | L1 ${ }_{l}$ | 0.00 | 0.75 | -107.03 | -98.03 | -9.00 | L1 ${ }_{l}$ | 0 | 0.56 | 0.00 | 0.75 |
| 210 | 0 | -51.13 | $L 3^{\text {B }}$ | 9 | $0.92{ }^{*}$ | -10.59 | $L 1_{e}$ | 0.00 | 0.38 | -68.44 | -51.13 | -17.32 | $L 3_{e}$ | 9 | 0.92 | 0.38 | 0.25 |
| 302 | 0 | -61.46 | $L 3^{\text {B }}$ | 7 | $1.11{ }^{* *}$ | -6.03 | $E q_{e}$ | 0.00 | 0.13 | -71.14 | -65.12 | -6.03 | $E q_{e}$ | 7 | 1.11 | 0.00 | 0.13 |
| 507 | 0 | -63.23 | L3 | 7 | $0.94 * *$ | - | - | - | - | - | - | - | - | - | - | - | - |
| 313 | 0 | -79.12 | $D 1^{\text {E }}$ | 2 | $2.68{ }^{* *}$ | -6.03 | $L 1_{e}{ }^{\ddagger}$ | 0.00 | 0.88 | -90.93 | -84.90 | -6.03 | $L 1_{e}^{\text {\#\# }}$ | 2 | 3.28 | 0.00 | 0.88 |
| 312 | 0 | -80.45 | $D 1^{\dagger}$ | 3 | 5.85** | -3.74 | $L 2_{e}{ }^{\ddagger}$ | 0.94 | 0.06 | -84.74 | -81.00 | -3.74 | $L 2_{e}$ | 3 | 1.37 | 0.94 | 0.06 |
| 204 | 2 | -84.86 | $D 1^{\text {E }}$ | 2 | $1.22{ }^{* *}$ | 0.00 | $L 1_{e}{ }^{\ddagger}$ | 0.00 | 1.00 | -88.47 | -88.47 | 0.00 | $L 1_{e}$ | 2 | 1.59 | 0.00 | 1.00 |
| 115 | 1 | -86.10 | D1 | 2 | $1.74 * *$ | -9.94 | $E q_{e}$ | 0.00 | 0.31 | -107.99 | -98.05 | -9.94 | $E q_{e}$ | 0 | 0.39 | 0.00 | 0.31 |
| 401 | 2 | -91.99 | $D 1^{\dagger}$ | 0 | $1.58{ }^{* *}$ | -6.03 | $E q_{l}$ | 0.00 | 0.13 | -104.35 | -98.32 | -6.03 | $E q_{l}$ | 0 | 0.32 | 0.00 | 0.13 |
| 310 | 0 | -41.69 | $E q^{\text {A }}$ | 11 | 0.00 | -9.94 | $L 1_{l}$ | 0.00 | 0.31 | -56.84 | -41.69 | -15.15 | $E q_{e l}$ | 11 | 0.00 | 0.13 | 0.31 |
| 315 | 0 | -41.80 | Eq | 11 | 0.00 | 0.00 | $L 3_{e}{ }^{\ddagger}$ | 1.00 | 0.00 | -50.80 | -41.80 | -9.00 | $E q_{e}$ | 11 | 0.00 | 0.00 | 0.75 |
| 404 | 1 | -54.69 | Eq | 9 | 0.03 | -9.00 | $E q_{e}{ }^{\ddagger}$ | 0.00 | 0.75 | -63.69 | -54.69 | -9.00 | $E q_{e}$ | 9 | 0.03 | 0.00 | 0.75 |
| 303 | 0 | -59.93 | Eq | 8 | 0.41 | -3.74 | $E q_{e}{ }^{\ddagger}$ | 0.00 | 0.06 | -63.68 | -59.93 | -3.74 | $E q_{e}$ | 8 | 0.41 | 0.00 | 0.06 |
| 417 | 0 | -60.52 | $E q^{\text {A }}$ | 8 | 0.30 | -10.97 | L1e | 0.00 | 0.44 | -73.80 | -60.52 | -13.29 | $E q_{e}$ | 8 | 0.30 | 0.31 | 0.63 |
| 202 | 0 | -60.78 | $E q^{\text {A }}$ | 8 | 0.10 | -9.94 | $E q_{e}$ | 0.00 | 0.31 | -70.72 | -60.78 | -9.94 | $E q_{e}$ | 8 | 0.10 | 0.00 | 0.31 |
| 518 | 0 | -66.38 | Eq | 7 | 0.61 | - | - | - | - | - | - | - | - | - | - | - | - |
| 112 | 2 | -66.39 | $E q$ | 7 | 0.00 | -16.64 | $L 2_{e}$ | 0.25 | 0.25 | -106.23 | -89.60 | -16.64 | $L 2_{e}$ | 3 | 0 | 0.25 | 0.25 |
| 215 | 0 | -73.85 | Eq | 6 | 0.55 | -3.74 | $L 1_{e}$ | 0.00 | 0.06 | -81.57 | -73.85 | -7.72 | $E q_{e}$ | 6 | 0.55 | 0.00 | 0.19 |
| 314 | 5 | -78.06 | Eq | 5 | 0.52 | -9.94 | $E q_{e}$ | 0.00 | 0.69 | -87.99 | -78.06 | -9.94 | $E q_{e}$ | 5 | 0.52 | 0.00 | 0.69 |
| 211 | 3 | -79.14 | Eq | 5 | 0.00 | -7.72 | $E q_{e}$ | 0.00 | 0.19 | -86.86 | -79.14 | -7.72 | $E q_{e}$ | 5 | 0.00 | 0.00 | 0.19 |
| 514 | 8 | -85.98 | Eq | 2 | 0.00 | - | - | - | - | - | - | - | - | - | - | - | - |
| 406 | 2 | -86.73 | Eq | 3 | 0.59 | -6.03 | $L 1_{l}$ | 0.00 | 0.13 | -99.17 | -86.73 | -12.44 | $E q_{l}$ | 3 | 0.59 | 0.06 | 0.25 |
| 212 | 5 | -96.62 | $E q^{\dagger}$ | 1 | 0.00 | -6.03 | $L 1_{e}$ | 0.00 | 0.88 | -104.34 | -96.62 | -7.72 | $E q_{e}$ | 1 | 0.00 | 0.00 | 0.81 |
| 506 | 0 | -82.10 | So | 3 | $1.26{ }^{* *}$ | - | - | - | - | - | - | - | - | - | - | - | - |
| 304 | 5 | -93.29 | $\mathrm{So}^{+}$ | 2 | 0.25 | 0.00 | $E q_{e}$ | 0.00 | 1.00 | -97.31 | -97.31 | 0.00 | $E q_{e}$ | 1 | 0 | 0.00 | 1.00 |
| 421 | 4 | -96.78 | $\mathrm{So}^{\dagger}$ | 1 | 0.31 | -10.59 | $E q_{e}$ | 0.00 | 0.38 | -109.34 | -98.38 | -10.97 | $L 1_{e}$ | 0 | 0.43 | 0.00 | 0.56 |

## Specification test and analysis

For some subjects these estimates leave room for doubt about whether our a priori specification of types omits relevant types and/or overfits by including irrelevant types

We conduct a subject by subject specification test that compares the likelihood of the subject's type estimate with those of estimates based on 88 pseudotypes, each constructed from one of our subject's guesses in the games

With regard to overfitting, for a subject's type estimate to be credible it should have higher likelihood than at least as many pseudotypes as at random: $87 / 8 \approx 11$ with i.i.d. likelihoods; estimated types that fail this test are marked "+" in Table IX

Now imagine that we had omitted a relevant type, say L2; the pseudotypes of subjects now estimated to be $L 2$ would then outperform the non-L2 types estimated for them, and would also make approximately the same (L2) guesses

Finding such a cluster we would diagnose an omitted type, and studying what its subjects' guesses have in common might help to reveal its decision rule; in Table IX possible clusters are identified by superscript letters A, B, C, D, or E

The next tables (from Appendix F) collect the guesses of the cluster candidates and summarize the games' structures; 310 and 409 are included as potential members of cluster A or E, respectively, despite some failures of the likelihood criteria

Game Structures, Types' Guesses, and Guesses of Cluster Candidates A, B, and C

| Game | ai | bi | pi | aj | bj | pi | L1 | L2 | L3 | D1 | D2 | E | S | 202 | 417 | (310) | 210 | 302 | 407 | 516 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 100 | 900 | 1.5 | 300 | 500 | 0.7 | 600 | 525 | 630 | 600 | 611.25 | 750 | 630 | 675 | 600 | 500 | 630 | 630 | 600 | 600 |
| 2 | 300 | 900 | 1.3 | 300 | 500 | 1.5 | 520 | 650 | 650 | 617.5 | 650 | 650 | 650 | 650 | 650 | 650 | 650 | 650 | 520 | 520 |
| 3 | 300 | 900 | 1.3 | 300 | 900 | 1.3 | 780 | 900 | 900 | 838.5 | 900 | 900 | 900 | 900 | 900 | 900 | 900 | 900 | 700 | 780 |
| 4 | 300 | 900 | 0.7 | 100 | 900 | 1.3 | 350 | 546 | 318.5 | 451.5 | 423.15 | 300 | 420 | 500 | 333.87 | 630 | 380 | 430 | 609 | 565 |
| 5 | 100 | 500 | 1.5 | 100 | 500 | 0.7 | 450 | 315 | 472.5 | 337.5 | 341.25 | 500 | 375 | 425 | 450 | 500 | 450 | 479 | 450 | 450 |
| 6 | 100 | 500 | 0.7 | 100 | 900 | 0.5 | 350 | 105 | 122.5 | 122.5 | 122.5 | 100 | 122 | 100 | 100 | 100 | 100 | 100 | 360 | 350 |
| 7 | 100 | 500 | 0.7 | 100 | 500 | 1.5 | 210 | 315 | 220.5 | 227.5 | 227.5 | 350 | 262 | 215 | 173.91 | 200 | 350 | 340 | 210 | 210 |
| 8 | 300 | 500 | 0.7 | 100 | 900 | 1.5 | 350 | 420 | 367.5 | 420 | 420 | 500 | 420 | 370 | 315 | 500/630 | 420 | 400 | 420 | 500 |
| 9 | 300 | 500 | 1.5 | 300 | 900 | 1.3 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500/999 | 500 | 500 |
| 10 | 300 | 500 | 0.7 | 100 | 900 | 0.5 | 350 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 490 |
| 11 | 100 | 500 | 1.5 | 100 | 900 | 0.5 | 500 | 225 | 375 | 262.5 | 262.5 | 150 | 300 | 310 | 300 | 500 | 375 | 370 | 225 | 225 |
| 12 | 300 | 900 | 1.3 | 300 | 900 | 1.3 | 780 | 900 | 900 | 838.5 | 900 | 900 | 900 | 900 | 900 | 900 | 900 | 900/999 | 900 | 692 |
| 13 | 100 | 900 | 1.3 | 300 | 900 | 0.7 | 780 | 455 | 709.8 | 604.5 | 604.5 | 390 | 695 | 600 | 520 | 400 | 550 | 555 | 455 | 455 |
| 14 | 100 | 900 | 0.5 | 300 | 500 | 0.7 | 200 | 175 | 150 | 200 | 150 | 150 | 162 | 150 | 150 | 150 | 150 | 160 | 210 | 175 |
| 15 | 100 | 900 | 0.5 | 100 | 500 | 0.7 | 150 | 175 | 100 | 150 | 100 | 100 | 132 | 100 | 100 | 100 | 100 | 100 | 175 | 175 |
| 16 | 100 | 900 | 0.5 | 100 | 500 | 1.5 | 150 | 250 | 112.5 | 162.5 | 131.25 | 100 | 187 | 240 | 227 | 100 | 187.5 | 218.75 | 250 | 375 |

Game Structures, Types' Guesses, and Guesses of Cluster Candidates D and E

| Game | ai | bi | pi | aj | bj | pj | L1 | L2 | L3 | D1 | D2 | E | S | 301 | 508 | 204 | 313 | (409) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 100 | 900 | 1.5 | 300 | 500 | 0.7 | 600 | 525 | 630 | 600 | 611.25 | 750 | 630 | 600 | 600 | 600 | 600 | 600 |
| 2 | 300 | 900 | 1.3 | 300 | 500 | 1.5 | 520 | 650 | 650 | 617.5 | 650 | 650 | 650 | 520 | 520 | 500 | 550 | 520 |
| 3 | 300 | 900 | 1.3 | 300 | 900 | 1.3 | 780 | 900 | 900 | 838.5 | 900 | 900 | 900 | 780 | 780 | 645 | 645 | 645 |
| 4 | 300 | 900 | 0.7 | 100 | 900 | 1.3 | 350 | 546 | 318.5 | 451.5 | 423.15 | 300 | 420 | 350 | 490 | 645 | 510 | 465 |
| 5 | 100 | 500 | 1.5 | 100 | 500 | 0.7 | 450 | 315 | 472.5 | 337.5 | 341.25 | 500 | 375 | 210 | 300 | 225 | 250 | 325 |
| 6 | 100 | 500 | 0.7 | 100 | 900 | 0.5 | 350 | 105 | 122.5 | 122.5 | 122.5 | 100 | 122 | 300 | 300 | 175 | 175 | 325 |
| 7 | 100 | 500 | 0.7 | 100 | 500 | 1.5 | 210 | 315 | 220.5 | 227.5 | 227.5 | 350 | 262 | 210 | 210 | 175 | 250 | 225 |
| 8 | 300 | 500 | 0.7 | 100 | 900 | 1.5 | 350 | 420 | 367.5 | 420 | 420 | 500 | 420 | 500 | 350 | 500/600 | 475 | 400 |
| 9 | 300 | 500 | 1.5 | 300 | 900 | 1.3 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500/520 | 500 | 500 | 475 | 475 |
| 10 | 300 | 500 | 0.7 | 100 | 900 | 0.5 | 350 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 400 |
| 11 | 100 | 500 | 1.5 | 100 | 900 | 0.5 | 500 | 225 | 375 | 262.5 | 262.5 | 150 | 300 | 150 | 340 | 150 | 200 | 325 |
| 12 | 300 | 900 | 1.3 | 300 | 900 | 1.3 | 780 | 900 | 900 | 838.5 | 900 | 900 | 900 | 780 | 780 | 645 | 645 | 645 |
| 13 | 100 | 900 | 1.3 | 300 | 900 | 0.7 | 780 | 455 | 709.8 | 604.5 | 604.5 | 390 | 695 | 350 | 430 | 645 | 510 | 645 |
| 14 | 100 | 900 | 0.5 | 300 | 500 | 0.7 | 200 | 175 | 150 | 200 | 150 | 150 | 162 | 200 | 200 | 300 | 200 | 200 |
| 15 | 100 | 900 | 0.5 | 100 | 500 | 0.7 | 150 | 175 | 100 | 150 | 100 | 100 | 132 | 150 | 150 | 175 | 175 | 150 |
| 16 | 100 | 900 | 0.5 | 100 | 500 | 1.5 | 150 | 250 | 112.5 | 162.5 | 131.25 | 100 | 187 | 150 | 150 | 175 | 175 | 175 |

A. Subjects 202, 310, and 417, all estimated Equilibrium: All made equilibrium guesses in exactly our $8 / 16$ games without mixed targets, and 310 also did so in 3 games with mixed targets; 202's and 417's deviations were always in the same direction but to different guesses, as were all but one of 310's (there is no pattern with respect to other aspects of structure)

We judge 202's and 417's guesses similar enough to meet the definition of a cluster, but we provisionally accept 310's identification as Equilibrium, which fits 310's guesses much better than 202's and 417's pseudotypes, despite similarities

However, the standard methods for identifying equilibrium guesses all work equally well with mixed targets, and only one of 29 Equilibrium R/TS subjects came close to 202's and 417's pattern; they may just have been using "homemade" rules that mimic Equilibrium in games without mixed targets
B. Subjects 210 and 302, both estimated $L 3$ with Equilibrium a fairly close second: Both deviate from $L 3$ in 7 games, 6 with mixed targets; and 302 also has minor deviations in 2 games, one with mixed targets (there is no other structural pattern)

6/7 of their common deviations are in the same direction, and all are to similar guesses; and both make exactly the equilibrium guess in game 6, the only game without mixed targets in which L3 and Equilibrium are separated

We judge these subjects' guesses similar enough for a cluster, but we cannot tell how they were determined; they may come from rules that are hybrids of $L 3$ and Equilibrium
C. Subjects 407, estimated L2; and 516, L1: Both make L1 guesses in most ( 5 or 7 ) of the first 9 games and $L 2$ guesses in most ( 6 or 4 ) of the last 7 (no other structural pattern)

These subjects' guesses are similar enough for a cluster, but we do not believe they followed an omitted type: the time pattern of deviations and the fact that most later guesses are more sophisticated suggest introspective learning; there are weak indications of an L1-L2 switch for a few other subjects
(By contrast, 108 made L2 guesses except for $L 1$ guesses in games 2, 10, and 16; most $L 1$ guesses are later and $L 2$ fits her/his guesses significantly better than any pseudotype)
D. Subjects 301 and 508, both estimated L1: Each of these subject's pseudotype is the only one with higher likelihood than the other's estimated type; they have five common deviations from L1, all downward but most to different guesses, and each also has one lone, upward deviation (no apparent structural pattern, but both lone deviations seem due to forgetting to multiply by own target, and some common deviations also seem due to cognitive errors such as forgetting or interchanging targets or limits)

These subjects' guesses are similar enough for a cluster, but we are not sure that they followed an omitted type; they may be sloppy L1s whose errors tended to fall in the same games
E. Subjects 204, estimated Equilibrium, 313, estimated L1, and 409, estimated L1: These subjects all made similar guesses, with 645s inexplicable by our types in (symmetric) games 3 and 12 and, for 204 and 409, in game 13 as well

Their guesses are similar enough for a cluster, but it is plain that they are not following a single omitted type

In their questionnaires all three stated homemade rules that depart from standard decision theory in different ways but, properly interpreted, explain most of their guesses exactly (Subject 409, for instance, says "...I took his/her lower limit and multiplied it by my target. If the resulting number was between my upper and lower limits, I kept that in mind. Otherwise I picked my lower limit. Then I took his/her upper limit and multiplied it by my target. Again, if the resulting number was within my range, I took it. Otherwise I picked the upper limit. Then I found the average of the two numbers." In symbols, 409 guessed $\left[\max \left\{a_{i}, a_{j} p_{i}\right\}+\min \left\{b_{i}, b_{j} p_{i}\right\}\right] / 2$. This rule explains her/his guesses exactly in 13/16 games.)

These subjects' homemade rules illustrate what we suspect is a common tendency for subjects to invent rules by which to process the data of games into decisions

To us it is less remarkable that these 3 subjects' rules deviate from standard decision theory than that most other subjects' homemade rules do conform to standard decision theory, even though most of them stop short of imposing equilibrium

## Summary of results for guesses (only)

Of the 43 subjects whose type estimate is $L 1,27$ are reliably identified as L1; the remaining 16 estimates may be spurious

Of the 20 subjects estimated to be $L 2,17$ are reliably identified; the remaining 3 estimates are probably spurious

Of the 3 subjects estimated to be L3, only one seems reliably identified; the other two estimates are probably spurious

Of the 5 subjects estimated to be D1, only one seems reliably identified; the other 4 estimates are probably spurious

Of the 14 subjects estimated to be Equilibrium, 11 seem reliably identified; the other 4 are probably spurious

Of the 3 subjects estimated to be Sophisticated, only one seems reliably identified; the other 2 are probably spurious

Thus, considering only guesses, 58 of our 88 subjects appear to be reliably identified as one of our types; almost all of them as L1, L2, or Equilibrium

These results are generally quite close to previous estimates from other kinds of games, except we find more Equilibrium subjects than most and no evidence of exotic types that (like SW's Worldly) respond to estimated population parameters

## Results for R/TS subjects' guesses

Table VII summarizes R/TS subjects' compliance with their assigned type's guesses in 16 games, and the failure rates in our second, type-specific understanding test

The results suggest that $L 1, L 2$, and $L 3$ are the easiest types to implement, with lower compliance for $L 1$ (due, we suspect, to subjects' attempts to outguess the computer)

Next highest in compliance is Equilibrium, still high enough that Baseline deviations from equilibrium are unlikely to be due primarily to cognitive limitations

Lowest in compliance are D1 and D2, although D1 and D2 failure rates are much lower than Equilibrium failure rates

Several D1 subjects (e.g. 804, with 3 D1 but 16 L2 guesses) made many more L2 than D1 guesses (after passing a D1 Understanding Test in which L2 answers were wrong), reinforcing the impression that $D k$ is less natural than $L k+1$

Table XIII. R/TS subjects' compliance with assigned type's guesses

|  | L1 | L2 | L3 | D1 | D2 | Eq. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCSD subjects | 7 | 9 | - | 11 | - | 10 |
| \% Compliance | 77.7 | 81.3 | - | 55.1 | - | 58.1 |
| \% Failed UT2 | 0.0 | 0.0 | - | 8.3 | - | 28.6 |
| York subjects | 18 | 18 | 18 | 19 | 19 | 19 |
| \% Compliance | 80.9 | 95.8 | 84.4 | 66.1 | 55.6 | 76.6 |
| \% Failed UT2 | 0.0 | 0.0 | 0.0 | 0.0 | 5.0 | 13.6 |
| UCSD+York subjects | 25 | 27 | 18 | 30 | 19 | 29 |
| \% Compliance | 80.0 | 91.0 | 84.7 | 62.1 | 55.6 | 70.3 |
| \% Failed UT2 | 0.0 | 0.0 | 0.0 | 3.2 | 5.0 | 19.4 |

## Results for Baseline and R/TS subjects' searches

Tables $X$ and XI give search data for R/TS subjects with high compliance with assigned type's guesses, and for Baseline subjects with high compliance with some type's guesses

R/TS subjects' look-up sequences are rich in their types' relevant sequences, as are the look-up sequences of Baseline subjects whose guesses conform closely to a type

Baseline subject 108, whose guesses switched for L2's to L1's in 3 games, gave no indication of the switches in his look-ups ( $L 2$ search automatically includes $L 1$ search); and subject 309, whose guesses coincided perfectly with L2's, made enough look-ups to be sure of identifying L2's guess only in games 6-16; he was just lucky in games 1-5

The econometric analysis of search quantifies compliance as the density of a type's relevant look-up sequence in the subject's sequence; Table IX reports estimates of subjects' types based on search only, and on guesses and search

Search-based estimates reaffirm the estimates based on guesses only for 51 of the 58 we argued were reliable; some estimates change because there is a tension between guesses and search; others because the subject did not satisfy the guesses-only type's search requirements

In the end 52 subjects are reliably identified: 27 as $\angle 1,13$ as L2, 10 as Equilibrium, and one each as L3 or Sophisticated (the last 2 were OB, and might not survive monitoring search)

## Table X. Selected R/TS Subjects' Information Searches and Assigned Types' Search Implications



Table XI. Selected Baseline Subjects' Information Searches and Estimated Types' Search Implications


