Assumptions

(P.I) $\mathcal{Y}^j$ is convex for each $j \in F$.
(P.II) $0 \in \mathcal{Y}^j$ for each $j \in F$.
(P.III) $\mathcal{Y}^j$ is closed for each $j \in F$.

The aggregate technology set is $Y = \sum_{j \in F} Y^j$.

(P.IV)(a) if $y \in Y$ and $y \neq 0$, then $y_k < 0$ for some k.
(b) if $y \in Y$ and $y \neq 0$, then $-y \notin Y$.
(P.V) For each $j \in F$, $\mathcal{Y}^j$ is strictly convex.
(P.VI) $\mathcal{Y}^j$ is a bounded set for each $j \in F$.

(C.I) X^i is closed and nonempty.
(C.II) $X^i \subseteq \mathbf{R}^N_+$. X^i is unbounded above, that is, for any $x \in X^i$ there is $y \in X^i$ so that $y > x$, that is, for $n = 1, 2, ..., N, y_n \geq x_n$ and $y \neq x$.
(C.III) X^i is convex.

(C.IV) (Non-Satiation) Let $x \in X^i$. Then there is $y \in X^i$ so that $y \succ_i x$.
(C.IV**) (Weak Desirability) X^i contains a translation of R^N_+. $x, y \in X^i$, $x >> y$(i.e. $x_n > y_n$, for all n) implies $x \succ_i y$.

(C.V) (Continuity) For every $x^\circ \in X^i$, the sets $A^i(x^\circ) = \{x \mid x \in X^i, x \succeq_i x^\circ\}$ and $G^i(x^\circ) = \{x \mid x \in X^i, x^\circ \succeq_i x\}$ are closed.

(C.VI)(WC) (Weak Convexity of Preferences) $x \succeq_i y$ implies $((1-\alpha)x+\alpha y) \succeq_i y$, for $0 \leq \alpha \leq 1$.
(C.VI)(SSC) (Semi-strict convexity of Preferences) $x \succ_i y$ implies $((1-\alpha)x + \alpha y) \succ_i y$, for $0 \leq \alpha < 1$.
(C.VI)(SC) (Strict Convexity of Preferences): Let $x \succeq_i y$, (note that this includes $x \sim_i y$), $x \neq y$, and let $0 < \alpha < 1$. Then $\alpha x + (1-\alpha)y \succ_i y$.

(C.VII) For all $i \in H$, $\tilde{M}^i(p) > \inf_{x \in X^i \cap \{x \mid |x| \leq c\}} p \cdot x$ for all $p \in P$.