FINAL EXAMINATION — Restatement of Problem 6

The restatement below changes some ambiguous plural pronouns to singular. Changes are **bolded**.

6. The Arrow Possibility Theorem can be stated in the following way. Let A be a set of alternative social choices (there are at least three distinct elements in A). Let P be the set of all transitive, reflexive preference orderings on A. Let P^K be the K-fold Cartesian product of P with itself. We take $\#K \geq 3$. A rational voting mechanism or 'Arrow Social Welfare Function' is then a mapping S, so that $S : P^K \to P$.

The Sen version of the Arrow Axioms can be stated:

Property 0) Universal Domain. The mapping really is from all of P^K.

Property 1) Non-dictatorship.

Property 2) Independence of Irrelevant Alternatives (only pairwise individual preferences matter in forming pairwise social preferences).

Property 3) Pareto principle (a universal preference is the social preference).

Then the Arrow Possibility Theorem can be stated as: There is no rational voting mechanism S fulfilling Properties 0,1, 2, and 3, for all A, P, K, as described above.

A Hare ballot (named after Lord Hare) for voting on a finite number, N, of alternatives can be described in the following way: Each voter ranks the alternatives and submits a ballot showing the ranking. In counting the ballots, the ballots are first arranged according to their top choices (denoted #1). Those alternatives receiving the larger number of voters' top choices remain in the running. **That one** receiving the smallest number (or 0) of top choices **is** out of the running (a tie-breaking rule may be needed). Ballots previously cast for the eliminated alternative are then redistributed among the remaining alternatives. Each is cast for its highest-ranking remaining alternative. Those alternatives with the larger number of ballots cast for them (on first or subsequent choices) remain in the running. **That one** with the smallest number cast for **it is** out of the running **(a tie breaking rule may be needed)**. Ballots cast for those eliminated are then redistributed as before. The process continues until the field is reduced to two alternatives. The remaining alternative attracting the majority of the ballots is chosen.

Evaluate the Hare balloting procedure in terms of the Sen version of the Arrow axioms. Does the procedure fulfill

a. Pareto Principle? Explain

b. Independence of Irrelevant Alternatives? Explain

c. Non-Dictatorship? Explain

d. Unrestricted Domain? Explain

e. Will voters find it advantageous to misstate their true preferences to influence the outcome (assuming they correctly anticipate other voters' ballots)? Explain