## Problem Set 3

**It's OK to work together on problem sets.**

*The passage below is adapted from a UCSD Economics Micro Qual. Hints and directions for you follow the statement of the problem.*

In the region of FleischundTrinken there are two perfectly divisible products: beer, y, and beef, x. The only factor of production is labor, L. There are 10,000 perfectly competitive households with perfectly divisible labor. They appear with the index i, j, k,(on the production side) and h (on the consumption side) below.

Beef production is described in the following way: Maximum possible beef production is limited by the available grazing land: for the whole region it is 1000 tons of beef. This is a static equilibrium problem: there are no conservation issues. Each (divisible) household has an unlimited supply of pet cattle that can be kept costlessly as household pets or sent (combined with (divisible) labor) out to graze on the pasture land for beef production. Each household j can devote labor of $0 \leq L^{x,j} \leq 1$ to beef production. Labor employed in beef raising by household j is denoted by $L^{x,j}$ and by household i (typically a dummy index) is $L^{x,i}$. All beef raising households have the same technology

$$x^j = L^{x,j}, \text{ when } \sum_{i=1}^{10,000} L^{x,i} \leq 1000$$

$$x^j = 1000\frac{\mathrm{L^{x,j}}}{\sum_{\mathrm{i=1}}^{10,000} \mathrm{L^{x,i}}}, \text{ when } \sum_{i=1}^{10,000} L^{x,i} > 1000$$

Household j treats the total labor applied to beef raising $\sum_{i=1}^{10,000} L^{x,i}$ parametrically. Household j does not take into account the effect of its own beef raising decision, $L^{x,j}$, on the total.

Beer production is described in the following way: Beer is produced under constant returns by many home-brew households, k, with the technology, $y^k = L^{y,k}$.

There are 10,000 laborers in FleischundTrinken, one per household, each endowed with one unit of (divisible) labor. The labor market-clearing condition is $L^{x,j} + L^{y,j} = 1$ for all $j = 1, ..., 10,000$. All households have the same utility function

$$u^h(C_x^h, C_y^h) = C_x^h + .5C_y^h$$

where $C_x^h$ denotes h's beef consumption, and $C_y^h$ denotes h's beer consumption. Leisure is not valued. Households sell their beer at the competitive price $p_y$ or sell the beef raised at the competitive price $p_x$; they may then repurchase the beer or beef at the same price. Set $p_y = 1$. That is, beer is the numeraire with price unity.

Find the following quantities determined in competitive general equilibrium, and explain how you derive them:

$$w = \text{competitive wage rate of labor}$$

$$\sum_{i=1}^{10{,}000} x^i = \text{total beef production/consumption}$$

$$\sum_{i=1}^{10{,}000} L^{x,i} = \text{total labor employed in beef raising}$$

$$p_x = \text{price of beef per ton}$$

$$\sum_{i=1}^{10{,}000} L^{y,i} = \text{total beer output}$$

The First Fundamental Theorem of Welfare Economics says that a competitive equilibrium allocation is Pareto efficient. Is that true of the the competitive equilibrium allocation you derived above?

If yes: explain fully, including the successful tradeoffs, marginal equivalences, etc.

If no: Find a Pareto preferable allocation and explain why the First Fundamental Theorem of Welfare Economics does not apply to this case.

*Assignment: Answer the adapted exam question.*

*Hints: You should be able to answer almost all of the question using the first order techniques in Starr's General Equilibrium Theory, Second Edition draft, Section 1.4 (the 2 × 2 × 2 model), particularly section 1.4.4. The correct answer is an interior solution (there is positive output of both beer and beef, positive labor employed in both lines of production) so the marginal value product of labor is equated between the two industries. In order to answer the Pareto efficiency question, draw a production possiblity set and see where the equilibrium output lies.*