Problem Set 1

1. Consider an Edgeworth Box chracterized by two households with identical utility functions. There are two commodities, x and y. Household 1 is charactered as

$$\begin{aligned} u^1(x,y) &= xy, \text{ with endowment} \\ r^1 &= (9,1). \end{aligned}$$

Household 2 is characterized as

$$\begin{aligned} u^2(x,y) &= xy, \text{ with endowment} \\ r^2 &= (1,9). \end{aligned}$$

a. Let prices be $p = (p_x, p_y) = (1/2, 1/2)$. Show that these are competitive equilibriumm rices resulting in the equilibrium allocation

$$\begin{aligned} (x^1, y^1) &= (5,5); \\ (x^2, y^2) &= (5,5); \end{aligned}$$

the superscripts denote the households.

b. Demonstrate that the allocation in (a) is Pareto efficient.

Discuss.

2. Using the Edgeworth Box specified in Problem 1

a. Describe fully the set of Pareto efficient allocations.

b. Describe fully the contract curve, the set of individually rational (that is, with individual utilities no worse than those of the endowment) Pareto efficient allocation.

3. Consider a particularly simple Robinson Crusoe (one household) economy. There is no labor or inputs to production. There are two goods, guavas, supplied in the quantity x, and oysters, supplied in the quantity y. There is a single firm producing the two outputs according to the production frontier described by

$$x^2 + y^2 = 100, \qquad x, y \geq 0. \tag{1}$$

Profits of the firm, Π, are Robinson's only source of income. The firm sells guavas for a price p^x and oysters for the price p^y with $p^x, p^y \geq 0$.

$$\Pi = p^x x + p^y y. \tag{2}$$

Robinson's household income then is

$$Y = \Pi. \tag{3}$$

His consumption of guavas is denoted g, and of oysters (shellfish) is denoted s. The household budget constraint is

$$Y = p^x g + p^y s. \tag{4}$$

a. Combine equations (2) - (4) to demonstrate the Walras' Law

$$0 = p^x(g - x) + p^y(s - y). \tag{5}$$

b. Describe (5) in words.

4. Consider an Edgeworth box (two households, A and B, two goods, x and y).

   Household A is characterized as:

   (a) endowment = (10, 0), ten units of x and zero of y;

   (b) $U^A(x^A, y^A) = x^A + 4y^A$; A likes y four times as much as A likes x.

   Household B is characterized as:

   (a) endowment = (0, 10), ten units of y and zero of x;

   (b) $U^B(x^B, y^B) = 5x^B + y^B$; B likes x five times as much as B likes y.

For both households, the two goods are perfect substitutes with MRS's respectively of (1/4) and 5.

(i) Draw an Edgeworth box for this economy. Show the endowment point, contract curve, competitive equilibrium (a) and the set of Pareto efficient points. Because of the linear preferences, the Pareto efficient set will not be a locus of smooth tangencies - - - don't bother differentiating anything. Show that $(x^A, y^A) = (0, 10)$, $(x^B, y^B) = (10, 0)$ is a competitive equilibrium.

(ii) Some writers would argue that:

the contract curve for this economy is equivalent to the set of competitive equilibria. That is, any individually rational Pareto efficient point in this Edgeworth box can be supported as a competitive equilibrium. These 'competitive equilibrium' allocations would include those of the form

$$\begin{aligned}(x^A, y^A), 2.5 &< y^A \leq 10, x^A = 0;\\ (x^B, y^B), x^B &= 10, y^B = 10 - y^A.\end{aligned}$$

Explain the reasoning for this argument (hint: think inside the box).

The assertion is false. Explain why it is mistaken (hint: think outside the box).

Problems 5, 6, and 7 work with an Edgeworth Box characterized by two households with identical utility functions. Superscripts are used to denote the name of the households. Households are characterized by a utility function and an endowment vector. There are two commodities, x and y.

Household 1 is characterized as

$$\begin{aligned}u^1(x^1, y^1) &= x^1y^1, \text{ with endowment;}\\ r^1 &= (8, 0).\end{aligned}$$

Household 2 is characterized as

$$\begin{aligned}u^2(x^2, y^2) &= x^2y^2, \text{ with endowment;}\\ r^2 &= (2, 10).\end{aligned}$$

A competitive equilibrium consists of prices $p^0 = (p^0_x, p^0_y)$ and allocation (x^{01}, y^{01}), (x^{02}, y^{02}) so that household 1's consumption plan (x^{01}, y^{01}) maximizes $u^1(x, y)$ subject to household 1's budget constraint, $p^0_x x + p^0_y y = 8p^0_x$, and similarly household 2's consumption plan (x^{02}, y^{02}) maximizes 2's utility subject to 2's budget, $p^0_x x + p^0_y y = 2p^0_x + 10p^0_y$ and so that markets clear:

$$(x^{01}, y^{01}) + (x^{02}, y^{02}) = (8, 0) + (2, 10) = (10, 10).$$

5. Let prices be $p = (p_x, p_y) = (1/2, 1/2)$. Show that these are competitive equilibrium prices resulting in the equilibrium allocation:

$$(x^1, y^1) = (4, 4); (x^2, y^2) = (6, 6).$$

6. Demonstrate that the allocation in 5 is Pareto efficient. It is not sufficient to cite the First Fundamental Theorem of Welfare Economics. The problem asks you to prove the theorem's result in this application. You may rely on the characterization of a Pareto efficient allocation in Starr's General Equilibrium Theory. Sufficient conditions for efficient allocation in an Edgeworth Box there (assuming concavity of utility functions) is equality of the MRS_{xy}'s.

7. The price system in problem 5, is said to 'decentralize' the allocation shown to be efficient in 6. Explain this notion of 'decentralize' or 'decentralization.'