MIDTERM EXAMINATION

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Ross or Aislinn — who promise to be clueless), until the examination is collected.

The exam is due at 1:30, in class, Thursday, February 12, 2009.

Answer any 4 (four) questions. An exam with five questions answered will be graded based on the lowest scoring four. The questions count equally.

All notation not otherwise defined is taken from Starr's General Equilibrium Theory, draft second edition. If you need to make additional assumptions to answer a question, that's OK. Do state the additional assumptions clearly.

Problems 1 and 2 deal with the following 2-commodity preferences.

Let there be two commodities x,y in the economy. The possible household consumption set is a translate of the nonnegative quadrant. $X^i \equiv \mathbf{R}^{\mathbf{2}}_{+} + \{(\mathbf{1}, \mathbf{1})\}$. All households have the same preferences $\succeq_i$ characterized in the following way:
$(x^\circ, y^\circ) \succ_i (x', y')$ if $x^\circ \cdot y^\circ > x' \cdot y'$, OR if $x^\circ \cdot y^\circ = x' \cdot y'$ and $x^\circ > x'$.
$(x^\circ, y^\circ) \sim_i (x', y')$ if $(x^\circ, y^\circ) = (x', y')$.

1. The preferences $\succeq_i$ do not fulfill C.V (Continuity) of Starr's *General Equilibrium Theory*. Give a mathematical demonstration of this property. A full proof is not required. What are the implications for demand behavior of the household?

2. Assume the economy with household preferences $\succeq_i$ fulfills all of the assumptions of Starr's *General Equilibrium Theory* Theorem 11.1, with the exception of C.V. In order to assure C.VII (Adequacy of Income), assume for all i, that $r^i \geq (2, 2)$ where the inequality holds co-ordinatewise.

In this economy, does there exist a competitive general equilibrium price vector? Give a 'yes' or 'no' or 'possibly but not always' answer and a mathematical demonstration of your answer. A full proof is not required.

3. Assume (without proof for the moment) that C.VI(WC) (weak convexity) and C.V (continuity) together imply C.VI(SSC) (semi-strict convexity).

Consider a proposed restatement of Starr's *General Equilibrium Theory* Theorem 14.1 (Equal Treatment in the Core), substituting C.VI(WC) (weak convexity) for C.VI(SC) (strict convexity).

Revised Theorem 14.1 (Equal treatment in the core with weak convexity) Assume C.IV**, C.V, and C.VI(WC). Let $\{x^{i,q}, i \in H, q = 1, \ldots, Q\}$ be in the core of Q-H, the Q-fold replica of economy H. Then for each $i, x^{i,q}$ is equally preferred for all q. That is, $x^{i,q} \sim_i x^{i,q'}$ for each $i \in H, q \neq q'$, where $\sim_i$ indicates indifference. That is, $x^{i,q} \succeq_i x^{i,q'}$, and $x^{i,q'} \succeq_i x^{i,q}$.

Is the proposed theorem true? Give a 'yes' or 'no' answer and a mathematical demonstration of your answer. A full proof is not required.

4. Consider the U-shaped cost curve model of the firm. This model is well formulated in Varian, *Microeconomic Analysis* (3rd edition), chapter 5. Is the U-shaped cost curve model consistent with the theory of production in Starr's *General Equilibrium Theory*, Theorem 11.1? In this setting are the conclusions of Theorem 11.1 true? Give answers of 'yes' or 'no' or 'possibly but not always' and a mathematical demonstration of your answer. A full proof is not required.

5. Consider the competitive equilibrium of an economy with two goods, x and y, and 400 identical (perfectly competitive) households. The utility of household i is described as $u^i(x^i, y^i; y^S) = x^i + \sqrt{y^i} - .1y^S$, where x^i is i's consumption of x, y^i is i's consumption of y, and y^S is the total volume of y supplied (generating a negative externality). Set p_x = price of x $\equiv 1$. p_y = price of y will be determined in a market equilibrium.

Relying on Ross's calculus (always a dicey proposition), household i's demand for y is $\frac{1}{4}p_y^{-2}$. Market demand for y is then $D^y(p_y) = 100p_y^{-2}$. Let the (assumed competitive) supply of y be $S^y(p_y) = 100 + 1000(p_y - 1)$ in the range $p_y \geq .9$.

Assume the market for y is in competitive equilibrium, with the market clearing level of y at y°. $y^\circ = S^y(p_y^\circ) = y^S = D^y(p_y^\circ)$. Introduce an excise tax α (per unit) on the supply of y, assessed on suppliers. Find $\frac{\partial \mathrm{p_y^\circ}}{\partial \alpha}$, and $\frac{\partial \mathrm{y}^\circ}{\partial \alpha}$. Is the tax paid primarily by buyers (in a price increased by the tax) or by sellers (in a price reduced, net of tax)? What is the effect on welfare, considering the externality from the supply of y? You may assume that tax receipts are rebated to households as $\frac{1}{400}$ of tax collections to each household, as a lump sum, treated parametrically.