Lecture Notes, March 10, 2009

Theory of the Second Best

Efficient allocation subject to specified (inefficient) distortions
- monopoly
- all available taxes are distortionary
- untreated externality

How to respond: optimize subject to the distortion. See Baumol and Bradford, "Optimal Departures from Marginal Cost Pricing" AER 1970. MasCollel et al, section 22.B.

Optimal taxation

Lump sum taxes --- no effect on incentives or efficiency of allocation.

Excise taxes
Special Case: All goods subject to excise taxation

The first order condition for Pareto efficiency is $MRS_{x,y} = p_x/p_y = MRT_{x,y}$
If all goods --- in all quantities --- are subject to excise taxation, then an efficient tax excise tax system leaves this equality unaffected at the margin. Hence a uniform proportionate (ad valorem) excise tax rate is efficient.

$(1+\tau)p_x/(1+\tau)p_y = MRS_{x,y} = p_x/p_y = MRT_{x,y}$

This is equivalent to a lump sum tax at the rate $\tau/(1+\tau)$ on all of endowment.

Distortionary Case: One untaxed good (e.g. leisure), several taxable goods. The general principle is to minimize the loss in Marshallian surplus (Consumer Surplus + Producer Surplus), known as *deadweight loss*, subject to achieving a target level of tax revenue. The general case includes cross elasticities of demand and elasticities of supply of the many goods. A conventional simplification is to ignore cross elasticities and elasticity of supply, considering only own-price elasticity of demand.

Special Case: Own-price elasticity of demand only (hold supply price constant, ignore cross elasticities of demand). This case is characterized by a diagonal Slutsky matrix $[\partial x_i/\partial p_j]$. Let ε_i be the own-price elasticity of demand for good i.

Note for example that when $\varepsilon_i = 0$ then the deadweight loss from an excise tax is zero --- there is no change in quantities demanded or produced, merely a transfer to the tax authority. The generalization is that the deadweight loss associated with a tax on good i is proportional to the own-price elasticity of the demand for i.

Ramsey (1927) optimal tax problem: Let t_i be the ad valorem rate of tax (percentage excise tax rate) on good i. Let good 0 be the untaxed good. Let CS(t) denote the total consumer surplus at tax rates $t = (t_1, t_2, ..., t_N)$.
We seek to

choose $t = (t_1, t_2, ..., t_N)$ to minimize $[CS(0) – CS(t)]$ subject to

$$\sum_{i=1}^{N} t_i p_i x_i = R$$

where R is the target level of revenue. The solution to this problem is the *Ramsey tax formula*, the *inverse elasticity rule*,

$$\frac{t_i}{t_j} = \frac{\varepsilon_j}{\varepsilon_i} \quad .$$

Proportional rates of excise taxation fall most heavily on goods with inelastic demand, since they display the smallest distortion in quantity demanded per unit revenue.

Topics: Debreu Mantel Sonnenschein
Anything goes for #H > #N
Regular Economies
Character of the Equilibrium Manifold
Local Uniqueness of equilibrium
Generic properties of economies

Debreu-Mantel-Sonnenschein
(e) $Z: P_+ \rightarrow R^N$
(c) Z is continuous
(W) $p \cdot Z(p) \leq 0$

D-M-S Theorem (MasColell et al, Thm 17.E.3) : For any Z satisfying (e), (c), and (W), there is a private ownership economy satisfying the usual closedness, convexity, and continuity conditions so that Z(p) is that economy's excess demand function.

Interpretation. The general equilibrium theory is uninformative. There are virtually no testable conditions on the excess demand function.

Brown and Matzkin: Testable implications of general equilibrium
Notation: subscripts denote households, superscripts denote differing states of the economy.
$\{(w, p)| Z(p, w)=0\}$ = equililibrium manifold $\subset R^{(\#H)N+N}$

where $w = (w_1, w_2, ..., w_{\#H})$ and w_i is houshold i's endowment in R^N_+ . x_i^1 is household i's consumption in state 1.

Then given individual data on
household changes in endowment
household changes in consumption
changes in general equilibrium prices
the consistency of the general equilibrium model can be tested using the principle of revealed preference.

Consider households 1 and 2 with endowments w_1, w_2 in R^N_+ . Let

$(w_1^1, w_2^1, p^1) \in$ equilibrium manifold (1)

$(w_1^2, w_2^2, p^2) \in$ equilibrium manifold (2)

This is a statement that is testable for consistency. For example suppose
$p^1 \cdot w_1^1 > p^1 \cdot (w_1^2 + w_2^2)$ (3), and
$p^2 \cdot w_1^2 > p^2 \cdot (w_1^1 + w_2^1)$ (4). Then we claim this is a contradiction: (1), (2), (3), (4) cannot all hold with these inequalities. Hence this is a testable restriction on the equilibrium manifold.

We have $p^2 \cdot x_1^1 < p^2 \cdot x_1^2$, $u_1(x_1^2) > u_1(x_1^1)$
and $p^1 \cdot x_1^2 < p^1 \cdot x_1^1$, $u_1(x_1^1) > u_1(x_1^2)$

This is a contradiction. (1), (2), (3), and (4) cannot occur simultaneously. Generalize this observation to many households, many endowments. This implies meaningful restrictions on the shape of the equilibrium manifold.

Objection: Too much individual data.

Reply: This is <u>really</u> general equilibrium. It includes p, w_i, x_i, and they all enter essentially.

Regular economies and the (local) uniqueness of equilibrium

Can there be multiple equilibria (isolated, locally unique)? Yes.

Can there be continua (infinite multiplicity) of equilibria? Yes, but this should be a rare event. How can we formalize the notion of "rare"?

Starr notation, $r^i \in R^N_+$ is household i's endowment.

Market equilibrium parameterized by endowment:

$Z(p; r^1, r^2, ..., r^{\#H}) = 0$. Normalize $p_N = 1$. Structure problem to avoid boundaries.

$Z: R^{N-1}_{++} \rightarrow R^{N-1}$

Apply the implicit function theorem. Define p^o by $Z(p^o; r^1, r^2, ..., r^{\#H}) = 0$. p^o is a function of $(r^1, r^2, ..., r^{\#H})$. Let the Jacobian of Z at $(p^o; r^1, r^2, ..., r^{\#H})$ be non-singular. Then p^o is locally unique.

"Generic" properties: properties that hold on an open dense subset of the domain.

Define $\mathcal{R}$ = set of regular economies

= $\{(r^1, r^2, ..., r^{\#H}) \in R^{\#HN}_{++} \mid$ Z has a non-singular Jacobian in equilibrium$\}$

= $\{(r^1, r^2, ..., r^{\#H}) \in R^{\#HN}_{++} \mid Z(p^o; r^1, r^2, ..., r^{\#H}) = 0$, p^o is locally unique$\}$

Assume u^i are smooth (twice continuously differentiable everywhere in R^N_+). Then $\mathcal{R}$ is open and dense in $R^{\#HN}_{++}$.

An economy in $\mathcal{R}$ has an odd number of equilibria.

Define $\mathcal{S}$ = set of singular economies = $R^{\#HN}_{++} \setminus \mathcal{R}$.

Sard's Theorem implies that $\mathcal{S}$ is closed and of Lebesgue measure 0.