Economics 200B Prof. R. Starr UCSD Winter 2009

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General equilibrium of the market economy with an excess demand correspondence; set-valued supply and demand

Supply correspondence with a weakly convex production technology: We now omit P.V and use P.I–P.IV only.

Let $S^j(p) = \{y^* | y^* \in Y^j, p \cdot y^* \geq p \cdot y \text{ for all } y \in Y^j\}$ be the supply correspondence of the firm.

Omit C.VI(SC), use C.VI(WC) only

$$D^i : \mathbf{P} \to \mathbf{R}^N,$$
$$D^i(p) \equiv \{y | y \in B^i(p) \cap X^i, y \succeq_i x \text{ for all } x \in B^i(p) \cap X^i\}$$
$$\equiv \{y | y \in B^i(p) \cap X^i, u^i(y) \geq u^i(x) \text{ for all } x \in B^i(p) \cap X^i\}.$$

The market economy

$$D(p) = \sum_{i \in H} D^i(p).$$

The economy's resource endowment is

$$r = \sum_{i \in H} r^i.$$

The supply side of the economy is characterized as

$$S(p) = \sum_{j \in F} S^j(p),$$

We can now summarize supply, demand, and endowment as an excess demand correspondence.

Definition The excess demand correspondence at prices $p \in P$ is $Z(p) \equiv D(p) - S(p) - \{r\}$.

Existence of competitive equilibrium

Definition $p^0 \in P$ is said to be a competitive equilibrium price vector if there is $z^0 \in Z(p^0)$ so that $z^0 \leq 0$ (coordinatewise) and $p^0_k = 0$ for k so that $z^0_k < 0$.

Theorem 17.7 Let the economy fulfill C.I - C.V, C.VI(WC), C.VII, and P.I–P.IV. Then there is a competitive equilibrium p^0 for the economy.

U-Shaped Cost Curves and Concentrated Preferences
The Non-Convex Economy

Omit P.I, C.VI(WC).

Artificial Convex Counterpart to the Non-Convex Economy
Convexified Technology and Supply

Starting from the non-convex technology set Y^j, we merely substitute its convex hull, $con(Y^j)$, for each $j \in F$, which trivially fulfills P.I. Then substitute the convex hull of the aggregate technology set for the aggregate set Y, $con(Y) = con(\sum_{j\in F} Y^j) = \sum_{j\in F} con(Y^j)$. Then we assume the convexified counterpart to P.III (the notation K is intended as a nmemonic for "convex")

PK.III $con(Y^j)$ is closed for all $j \in F$.

The economic implication of PK.III is that scale economies are bounded — as in the U-shaped cost curve case; average costs are not indefinitely diminishing.

PK.IV (a) if $y \in con(Y)$ and $y \neq 0$, then $y_k < 0$ for some k.

(b) if $y \in con(Y)$ and $y \neq 0$, then $-y \notin con(Y)$.

$S^{kj}(p) \equiv \{y^o \in con(Y^j) | p \cdot y^o \geq p \cdot y \text{ for all } y \in con(Y^j)\}$.

The artificially convex profit function of firm j is

$\pi^{kj}(p) \equiv p \cdot y^o$, where $y^o \in S^{kj}(p)$.

Under PK.III, a typical point of $S^{kj}(p)$ will be a point of $S^j(p)$ or a convex combination of points of $S^j(p)$.

Lemma: Assume P.II, PK.III, PK.IV and suppose $S^{kj}(p)$ is nonempty (exists and is well defined). Then $y^j \in S^{kj}(p)$ implies $y^j \in con(S^j(p))$ and $\pi^{kj}(p) = \pi^j(p)$.

Artificial Convex Preferences and Demand

Definition: Let $x, y \in X^i$. We say $x \succeq_{ki} y$ if for every $w \in X^i$, $y \in con(A^i(w))$ implies $x \in con(A^i(w))$.

(CK.IV) For each $i \in H$, $\succeq_{ki}$ fulfills C.IV.

(CK.V) For each $i \in H$, $\succeq_{ki}$ fulfills C.V.

(CK.VI) For each $i \in H$, $\succeq_{ki}$ fulfills C.VI(WC).

$D^{kh}(p) \equiv \{x^o | x^o \in B^h(p), x^o \succeq_{kh} x \text{ for all } x \in B^h(p)\}$.

Lemma: Assume C.I, C.II, C.III,CK.IV, CK.V, CK.VI, C.VII. Let $p \in P$, and suppose $M^i(p)$ exists. Then $x^i \in D^{ki}(p)$ implies $x^i \in con(D^i(p))$.

Competitive Equilibrium in the Artificial Convex Economy

Firms: $j \in F$, technologies are $con(Y^j)$, fulfilling P.I (trivally), P.II, PK.III, PK.IV.

Households: $i \in H$, tastes $\succeq_{ki}$, fulfilling C.I, C.II, C.III,CK.IV, CK.V, CK.VI, C.VII; endowments r^i, firm shares α^{ij}.

Then this economy fulfills all of the assumptions of Theorem 17.7. Applying that theorem, we know the convex economy has a general competitive equilibrium. That is,

Lemma: Assume P.II, PK.III, PK.IV, C.I, C.II, C.III,CK.IV, CK.V, CK.VI, C.VII. Then there are prices $p^o \in P$, production plans $y^{oj} \in S^{kj}(p^o)$, consumption plans $x^{oi} \in D^{ki}(p^o)$ so that markets clear

$$\sum_{i \in H} r^i + \sum_{j \in F} y^{oj} \geq \sum_{i \in H} x^{oi}$$

where the inequality applies co-ordinatewise, and $p^o_n = 0$ for n so that the strict inequality holds.

Approximate Equilibrium

Definition: We define the inner radius of $S \subset R^N$ as

$r(S) \equiv \sup_{x \in con(S)} \inf_{T \subset S; x \in con(T)} rad(T)$

Corollary to the Shapley-Folkman Theorem: Let F be a finite family of compact subsets $S \subset R^N$ and $L > 0$ so that $r(S) \leq L$ for all $S \in F$. Then for any $x \in con(\sum_{S \in F} S)$ there is $y \in \sum_{S \in F} S$ so that $|x - y| \leq L\sqrt{N}$.

Theorem 18.1 Let the economy fulfill P.II, PK.III, PK.IV. and C.I, C.II, C.III, CK.IV, CK.V, CK.VI, C.VII. Let there be $L > 0$ so that for all $h \in H$, $x \in X^h$, $f \in F$,

$$r(A^h(x)) \leq L, \text{ and } r(Y^f) \leq L.$$

Then there are prices $p^* \in P$, production plans $y^{\dagger j} \in Y^j$, $y^{*j} \in con(Y^j)$, consumption plans $x^{*i} \in X^i$, and $x^{\dagger i} \in X^i$ so that

$$\sum_{i \in H} x^{*i} \leq \sum_{j \in F} y^{*j} + r$$

$$p^*_k = 0 \text{ for } k \text{ so that } \sum_{i \in H} x^{*i}_k < \sum_{j \in F} y^{*j}_k + r_k$$

$$p^* \cdot x^{\dagger i} = p^* \cdot r^i + \sum_{j \in F} \alpha^{ij} p^* \cdot y^{\dagger j} = p^* \cdot r^i + \sum_{j \in F} \alpha^{ij} p^* \cdot y^{*j}$$

$$x^{\dagger i} \text{ maximizes } u^i(x) \text{ subject to } p^* \cdot x \leq p^* \cdot r^i + \sum_{j \in F} \alpha^{ij} p^* \cdot y^{\dagger j}$$

$$|[\sum_{i \in H} x^{*i} - \sum_{j \in F} y^{*j}] - [\sum_{i \in H} x^{\dagger i} - \sum_{j \in F} y^{\dagger j}]| \leq L\sqrt{N}$$

Proof By the lemma of section 18.3.3 there is $p^* \in P$, $y^{*j} \in S^{kj}(p^*)$, $x^{*i} \in D^{ki}(p^*)$ so that

$\sum_{i \in H} r^i + \sum_{j \in F} y^{*j} \geq \sum_{i \in H} x^{*i}$, with $p^*_k = 0$ for k so that a strict inequality holds,

and $p^* \cdot x^{*i} = p^* \cdot r^i + \sum_{j \in F} \alpha^{ij} p^* \cdot y^{*j}$.

Using the lemmata , $y^{*j} \in con(S^j(p^*))$ and $x^{*i} \in con(D^i(p^*))$. Applying the Corollary to the Shapley-Folkman Theorem, for each $j \in F$ there is $y^{\dagger j} \in S^j(p^*)$, and for each $i \in H$ there is $x^{\dagger i} \in D^i(p^*)$ so that

$$|[\sum_{i \in H} x^{*i} - \sum_{j \in F} y^{*j}] - [\sum_{i \in H} x^{\dagger i} - \sum_{j \in F} y^{\dagger j}]| \leq L\sqrt{N}$$

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QED