Lecture Notes, January 13, 2008

Partial equilibrium comparative statics

Partial equilibrium: Market for one good only with supply and demand as a function of price. Price is defined as the solution to the equation.

$$z(p) = D(p) - S(p) = 0$$

The implicit assumption is *ceteris paribus*, other things being equal (all other prices held fixed). Suppose there is a shift parameter, α , that describes changes in the demand and supply functions. Then the definition of equilibrium now looks like:

$$z(p,\alpha) = D(p,\alpha) - S(p,\alpha) = 0 .$$

Consider changes in α. What happens to p?

Totally differentiate z with respect to α. We have

$$\frac{dz}{d\alpha} = \frac{\partial z}{\partial p}\frac{dp}{d\alpha} + \frac{\partial z}{\partial \alpha} = 0$$

Assuming $\frac{\partial z}{\partial p} \neq 0$ we have

$$\frac{dp}{d\alpha} = -\left(\frac{1}{\frac{\partial z}{\partial p}}\right)\frac{\partial z}{\partial \alpha} = -\frac{\frac{\partial z}{\partial \alpha}}{\frac{\partial z}{\partial p}} = -\frac{D_\alpha - S_\alpha}{D_p - S_p}$$

(the denominator $\frac{\partial z}{\partial p} = D_p - S_p$ of this expression is the Jacobian of the system).

Then suppose that α represents an upward shift in demand and that the usual slopes apply to D and S. $D_p < 0, S_p > 0$. We have

$$\frac{dp}{d\alpha} = -\frac{(+)-(0)}{(-)-(+)} = -\left(\frac{+}{-}\right) = +$$

Just what you'd expect. An upward shift in demand increases price.

Example: The tax incidence problem

Who really pays a tax levied on sellers?

Let α = excise tax, p^o - α= price received by seller, p^o = price paid by buyer

D (p, α) = D (p , 0) , S(p, α) = S(p - α, 0)

D_α (p, α) = 0 , S_α(p, α) = – S_p .

$$\frac{dp^o}{d\alpha} = -\frac{D_\alpha - S_\alpha}{D_p - S_p} = -\frac{-(-S_p)}{D_p - S_p} = \frac{-S_p}{D_p - S_p}$$

Consider the case $S_p >> 0$, $D_p \approx 0$; elastic supply, inelastic demand.

Then $\frac{dp^o}{d\alpha} \approx 1$. Interpretation: Price to seller is unaffected by imposition of the tax α . The tax is shifted to buyers.

Comparative Statics, Implicit Function Theorem

Characterize market equilibrium, subject to a parameter α, by market clearing in z (p, α).

$$\begin{pmatrix} z_1(p,\alpha) \\ \vdots \\ z_i(p,\alpha) \\ \vdots \\ z_N(p,\alpha) \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Prices p are endogenously determined by the market clearing condition. Then as α shifts, market-clearing values of p will change as well. Assuming everything in sight is differentiable and well defined, we have,

$$\begin{pmatrix} \frac{dz_1}{d\alpha} \\ \vdots \\ \vdots \\ \vdots \\ \frac{dz_N}{d\alpha} \end{pmatrix} = \begin{pmatrix} \frac{\partial z_1}{\partial p_1} & & & \frac{\partial z_1}{\partial p_N} \\ & \ddots & & \\ \vdots & & \frac{\partial z_i}{\partial p_j} & \vdots \\ & & \ddots & \\ \frac{\partial z_N}{\partial p_1} & & & \frac{\partial z_N}{\partial p_N} \end{pmatrix} \begin{pmatrix} \frac{dp_1}{d\alpha} \\ \vdots \\ \frac{dp_i}{d\alpha} \\ \vdots \\ \frac{dp_N}{d\alpha} \end{pmatrix} + \begin{pmatrix} \frac{\partial z_1}{\partial \alpha} \\ \vdots \\ \frac{\partial z_i}{\partial \alpha} \\ \vdots \\ \frac{\partial z_N}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The expression $\begin{pmatrix} \frac{\partial z_1}{\partial p_1} & & & \frac{\partial z_1}{\partial p_N} \\ & \ddots & & \\ \vdots & & \frac{\partial z_i}{\partial p_j} & \vdots \\ & & \ddots & \\ \frac{\partial z_N}{\partial p_1} & & & \frac{\partial z_N}{\partial p_N} \end{pmatrix}$ is the ***Jacobian*** of the market clearing equation system. Solving for $\begin{pmatrix} \frac{dp_1}{d\alpha} \\ \vdots \\ \frac{dp_i}{d\alpha} \\ \vdots \\ \frac{dp_N}{d\alpha} \end{pmatrix}$ we have

$$\begin{pmatrix} \frac{dp_1}{d\alpha} \\ \vdots \\ \frac{dp_i}{d\alpha} \\ \vdots \\ \frac{dp_N}{d\alpha} \end{pmatrix} = -\begin{pmatrix} \frac{\partial z_1}{\partial p_1} & & & \frac{\partial z_1}{\partial p_N} \\ & \ddots & & \\ \vdots & & \frac{\partial z_i}{\partial p_j} & \vdots \\ & & \ddots & \\ \frac{\partial z_N}{\partial p_1} & & & \frac{\partial z_N}{\partial p_N} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial z_1}{\partial \alpha} \\ \vdots \\ \frac{\partial z_i}{\partial \alpha} \\ \vdots \\ \frac{\partial z_N}{\partial \alpha} \end{pmatrix}$$

This expression is well defined when the Jacobian is non-singular. This is an application of the **Implicit Function Theorem.**

See also **Regular Economies**.

HELP: There is a shortage of good, intelligent, relatively simple transparent, comparative statics problems suitable for a problem set or exam. Please suggest your favorite to Ross. Reward: 3 brownie points plus your question may show up where it will do you the most good.