FINAL EXAMINATION

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Ross or Aislinn — who promise to be clueless), until the examination is collected.

The exam is due by 11:45 AM, Thursday, March 19, 2009. Turn in to the attendant in room 245 Sequoyah or e-mail to abohren@weber.ucsd.edu

Answer any 5 (five) questions. An exam with more than five questions answered will be graded based on the lowest scoring five. The questions count equally.

All notation not otherwise defined is taken from Starr's General Equilibrium Theory, draft second edition. If you need to make additional assumptions to answer a question, that's OK. Do state the additional assumptions clearly.

1. Consider the following example of supply and demand relations between two markets. There are two goods, denoted 1 and 2, with prices p_1 and p_2, supply functions $S_1(p_1, p_2)$ and $S_2(p_1, p_2)$, and demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$. These are specified by the expressions

$$S_1(p_1, p_2) = 3p_1; \qquad D_1(p_1, p_2) = 8 - 4p_2 - p_1; p_2 \leq 2$$

and

$$S_2(p_1, p_2) = 5p_2; \qquad D_2(p_1, p_2) = 24 - 6p_1 - p_2; p_1 \leq 4.$$

The market for good 1 is said to be in equilibrium at prices (p_1^o, p_2^o) where $S_1(p_1^o, p_2^o) = D_1(p_1^o, p_2^o)$. The market for good 2 is said to be in equilibrium at prices (p'_1, p'_2) where $S_2(p'_1, p'_2) = D_2(p'_1, p'_2)$. Demonstrate that each market has an equilibrium when the other's price is fixed. Show that, nevertheless, no pair of prices exists for the two markets at which they are both in equilibrium. Does this supply-demand system provide a counterexample to Theorems 1.2, 7.1, 11.1, 17.7, on the existence of general equilibrium prices? Explain fully.

2. *(This problem is in a familiar setting, Econ 200B Winter 2009, Problem Set 7. Use the definitions that appear there of competitive equilibrium and the core. What sets this problem apart is the variation among the households — they have different utility functions, depending on the values of the parameters Υ^i and k^i.)*

Consider an economy of a thousand (1000) households $i \in H$, a finite set of firms F, and two commodities known as x and g. Each household i, is endowed with $\bar{X}^i$ of good x. Good g is produced by firms $j \in F$ (all of which have the same constant returns technology), at the rate of one unit of output g for each unit of input x. g^i denotes household i's purchase of good g.

We define $G = \sum_{i \in H} g^i$ (1)

Let each i have a continuous weakly concave utility function

$u^i(x^i, G) = x^i + k^i \min[G, \Upsilon^i]$, for $G \geq 0$, where x^i may be ≤ 0 or ≥ 0 (this is MasColell et al's quasi-linear model) , $k^i \geq 0, \Upsilon^i \geq 0$. **The values Υ^i, k^i vary with i. You are asked to specify them in special cases below.**

That is, household i enjoys G up to a maximum of Υ^i and likes each unit of G k^i as much as he likes x. g^i and G are public goods. The utility function is continuous everywhere, but it is not differentiable with respect to G in the neighborhood of $G = \Upsilon^i$.

We'd like to review the competitive equilibrium allocation in this setting. Assume marginal cost pricing: the price of x equals the price of g and we can set these prices at unity, $p_x = 1 = p_g$, for convenience. All firms run zero profits so household income is merely the value of endowment. We maintain the convention that households sell all of endowment and repurchase the amount they wish to consume.

Household i's budget constraint in a marginal cost pricing equilibrium reads

$x^i + g^i = \bar{X}^i$ (2)

where x^i is i's purchase of good x, and g^i is i's purchase of good g (good x acts as numeraire). Recall that x^i may be negative or positive. Household i's competitive market consumption choice problem is to

Choose x^i, g^i, to maximize $u^i(x^i, g^i + \sum_{h \in H, h \neq i} g^h)$ (3)

subject to (2).

Household i treats the prices of x and g parametrically and treats the choices of g^h of other households, $h \neq i$, parametrically as well. We define a competitive equilibrium for this economy as choices $x^{*i}, g^{*i}, G^* = \sum_{i \in H} g^{*i}$, fulfilling (2) and (3) for each household i so that all markets clear, that is, so that $G^* + \sum_{i \in H} x^{*i} = \sum_{i \in H} \bar{X}^i$.

(part A) Will a competitive equilibrium typically exist in this model? Explain.

(part B) Find values of Υ^i, k^i so that a competitive equilibrium allocation — if it exists — will be Pareto inefficient. Are these values unique? Explain.

(part C) Is it possible to find values of Υ^i, k^i so that a competitive equilibrium allocation — if it exists — will be Pareto efficient? If so find them. Are these values unique? Explain.

3. Same setting as question 2. Use the definition of the core that appears in Econ 200B Winter 2009, Problem Set 7.

(part A) Will a non-empty core typically exist in this model? Explain.

(part B) If the core is non-empty, will the core allocation be Pareto efficient? Explain.

(part C) Are there values of Υ^i, k^i so that a core allocation exists and is Pareto efficient? If so find them. Are these values unique? Explain.

4. Prof. Robert Clower (Northwestern, UCLA, University of South Carolina) comments: "Walrasian analysis is limited strictly to convex economies," (in "Economics as a deductive science," *Southern Economic Journal*, 1994). Evaluate this comment with regard to Chapters 1 - 11, Theorem 14.3 and Theorem 18.1 of Starr's *General Equilibrium Theory*, draft second edition.

Is the statement right always, never, sometimes?

Do Theorems 14.3 and 18.1 require convexity of preferences or technology?

Does Theorem 18.1 treat the case of natural monopoly — unbounded scale economy (unbounded nonconvexity)?

Does Theorem 14.3 imply that the First Fundamental Theorem of Welfare Economics is true in a large economy of nonconvex preferences?

5. Consider a perfectly competitive economy with external effects. There are a thousand (1000) households $i \in H$, a finite set of firms F, and three commodities known as x, y, and z (z does **not** stand for excess demand). Each household i, is endowed with $\bar{X}^i > 3$ of good x. Goods y and z are produced by firms $j \in F$ (all of which have the same constant returns technology), at the rate of one unit of output of y or z for each unit of input x. We take general equilibrium prices of x, y, z, then to be $p = (p_x, p_y, p_z) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ We define $Z = \sum_{i \in H} z^i$, where z^i is household i's purchase of z . Each household i has a parameter $0 \leq k^i \leq 1$ and the value of k^i varies across households. Let $[k^i Z]$ denote the biggest integer less than or equal to $k^i Z$, $(\leq k^i Z)$. Household utility functions are of the form

$u^i(x, y, z; Z) = x^{\frac{1}{3}}$ when $[k^i Z]$ is evenly divisible by 2 but not by 3,

$u^i(x, y, z; Z) = (x + y)^{\frac{1}{2}}$ when $[k^i Z]$ is evenly divisible by 3 but not by 2,

$u^i(x, y, z; Z) = x + y + z$ otherwise.

part A Is there a competitive equilibrium allocation in this economy? Acceptable answers are "yes", "no", "possibly, but not always". Explain.

part B If there exists a competitive equilibrium price and allocation, is the allocation Pareto efficient? Acceptable answers are "yes", "no", "possibly, but not always", "yes, in a rather unsatisfactory way". Explain.

6. The Arrow Possibility Theorem can be stated in the following way. Let A be a set of alternative social choices (there are at least three distinct elements in A). Let P be the set of all transitive, reflexive preference orderings on A. Let P^K be the K-fold Cartesian product of P with itself. We take $\#K \geq 3$. A rational voting mechanism or 'Arrow Social Welfare Function' is then a mapping S, so that $S : P^K \to P$.

The Sen version of the Arrow Axioms can be stated:

Property 0) Universal Domain. The mapping really is from all of P^K.

Property 1) Non-dictatorship.

Property 2) Independence of Irrelevant Alternatives (only pairwise individual preferences matter in forming pairwise social preferences).

Property 3) Pareto principle (a universal preference is the social preference).

Then the Arrow Possibility Theorem can be stated as: There is no rational voting mechanism S fulfilling Properties 0,1, 2, and 3, for all A, P, K, as described above.

A Hare ballot (named after Lord Hare) for voting on a finite number, N, of alternatives can be described in the following way: Each voter ranks the alternatives and submits a ballot showing the ranking. In counting the ballots, the ballots are first arranged according to their top choices (denoted #1). Those alternatives receiving the larger number of voters' top choices remain in the running. Those receiving the smallest number (or 0) of top choices are out of the running (a tie-breaking rule may be needed). Ballots previously cast for one of the eliminated alternatives are then redistributed among the remaining alternatives. Each is cast for its highest-ranking remaining alternative. Those alternatives with the larger number of ballots cast for them (on first or subsequent choices) remain in the running. Those with the smallest number cast for them are out of the running. Ballots cast for those eliminated are then redistributed as before. The process continues until the field is reduced to two alternatives. The remaining alternative attracting the majority of the ballots is chosen.

Evaluate the Hare balloting procedure in terms of the Sen version of the Arrow axioms. Does the procedure fulfill

a. Pareto Principle? Explain

b. Independence of Irrelevant Alternatives? Explain

c. Non-Dictatorship? Explain

d. Unrestricted Domain? Explain

e. Will voters find it advantageous to misstate their true preferences to influence the outcome (assuming they correctly anticipate other voters' ballots)? Explain

7. A practical problem facing the United States economy (and other market economies) at the present time is the collapse of a real estate speculative bubble. Putting that in the context of an Arrow-Debreu economy under uncertainty with a full set of contingent commodity markets:

Some households purchased housing services far into the future with the expectation of events occurring so that they would have highly valuable holdings in the event that occurred. Their expectations were mistaken. They financed these purchases by the sale of endowment in events that they thought would not occur, so that they would not have to deliver the promised sales. Indeed, they sold contracts on goods they did not have and cannot deliver (bankruptcy or default).

Can this "bubble collapse" event occur in an Arrow-Debreu economy with a full set of contingent commodity contracts in equilibrium ? Can household expectations be disappointed? Can bankruptcy or default occur? Explain.