

### Problem Set 1 — Suggested Answer <sup>1</sup>

**It's OK to work together on problem sets.**

*The passage below is taken verbatim from the UCSD Economics Micro Qual, June 2007. Hints and directions for you follow the statement of the problem.*

On the island of Vinopesce there are two perfectly divisible products: wine,  $y$ , and fish,  $x$ . The only factor of production is labor,  $L$ . Maximum possible fish catch for the whole island is 100 fish. This is a static equilibrium problem: there are no conservation issues. There are ten perfectly competitive fishing firms, denoted  $j = 1, 2, \dots, 10$ . Labor employed by firm  $j$  is denoted by  $L^j$  and by firm  $i$  (typically a dummy index) is  $L^i$ . All firms have the same technology

$$x^j = L^j, \text{ when } \sum_{i=1}^{10} L^i \leq 100$$

$$x^j = 100 \frac{L^j}{\sum_{i=1}^{10} L^i}, \text{ when } \sum_{i=1}^{10} L^i > 100$$

Wine is produced under constant returns by many firms  $k$ , with the technology,  $y^k = L^k$ . There are 1000 laborers on Vinopesce, one per household, each endowed with one unit of (divisible) labor. All households have the same utility function

$$u^h(x^h, y^h) = x^h + .5y^h$$

where  $x^h$  denotes  $h$ 's fish consumption, and  $y^h$  denotes  $h$ 's wine consumption. Leisure is not valued. Households sell their labor at the competitive wage rate  $w$ . Set  $p_y = 1$ . That is, wine is the numeraire with price unity. Find the following quantities determined in competitive equilibrium, and explain how you derive them:

$w$  = competitive wage rate of labor

$$\sum_{i=1}^{10} x^i = \text{total fish harvest}$$

$$\sum_{i=1}^{10} L^i = \text{total labor employed in fishing}$$

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$p_x$  = price of fish

total wine output

The First Fundamental Theorem of Welfare Economics says that a competitive equilibrium allocation is Pareto efficient. Is that true of the the competitive equilibrium allocation you derived above?

If yes: explain fully, including the successful tradeoffs, marginal equivalences, etc.

If no: Find a Pareto preferable allocation and explain why the First Fundamental Theorem of Welfare Economics does not apply to this case. Find a tax/subsidy or Lindahl pricing scheme that will achieve Pareto efficient allocation. Explain.

*Assignment: Answer the exam question.*

*Hints: You should be able to answer almost all of the question using the first order techniques in Starr's General Equilibrium Theory, Second Edition draft, Section 1.4 (the  $2 \times 2 \times 2$  model), particularly section 1.4.4. The correct answer is an interior solution (there is positive output of both wine and fish) so the marginal value product of labor is equated between the two industries. In order to answer the Pareto efficiency question, draw a production possibility set and see where the equilibrium output lies. We haven't yet studied tax/subsidy Lindahl pricing in class, so if it's not obvious, don't bother finding a tax/subsidy plan (if it is indeed needed).*

## Suggested Answer

### (a) Quantities in competitive equilibrium

Assume an interior solution. Then we have that the marginal value product of labor is equated in both fish and wine and equals the wage rate. We have

marginal value product of labor in wine =  $p_y \times$  marginal product of labor in wine =  $p_y \times 1 = w = 1$

Marginal rate of substitution of fish for wine is  $2 = p_x/p_y$ , so  $p_x = 2$ .

**There are two ways to answer this question, myopic and far-sighted. The myopic answer treats the total level of labor devoted to fishing parametrically. That's appropriate if the number of fishing firms is large and each firm's additional labor in fishing is negligible. It's an assumption of perfect competition. Then each firm treats it's marginal product of labor as locally constant. Here's the myopic answer.**

$1 =$  marginal value product of labor in wine = marginal value product of labor in fishing  $\approx p_x \times \min[1, 100 \frac{1}{\sum_1^{10} L^i}] = 2 \times \min[1, 100 \frac{1}{\sum_1^{10} L^i}]$  so  $\sum_1^{10} L^i = 200$ .

Then total fish harvest  $\sum_1^{10} x^i = \sum_{i=1}^{10} 100 \frac{L^i}{\sum_{i=1}^{10} L^i} = 100$ .

Labor is fully employed. 1000 laborers - 200 in fishing = 800 laborers in wine, so total wine output = 800. The question does not directly ask for distribution of income but it is useful to know: wine and fish generate 0 - profit. All net income is the wages of labor, equal for all households = 1. The utility of the typical household is 0.50.

**The far-sighted answer allows each firm to recognize that its marginal product of labor is not locally constant. It recognizes its own effect on the denominator  $\sum_{i=1}^{10} L^i$  on its own marginal product (not taking into account the effect on other firms). Here's the far-sighted answer.**

$$x^j = 100 \frac{L^j}{\sum_{i=1}^{10} L^i}, \text{ when } \sum_{i=1}^{10} L^i > 100$$

Recall the chain rule for fractions:  $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$ . For simplicity assume the same value of  $L^i$  for all  $i$ ,  $\bar{L}$ . Then

$\frac{dx^j}{dL^j} = \frac{100 \sum L^i - (100L^j)}{(\sum L^i)^2} = \frac{100(10\bar{L}) - (100\bar{L})}{(10\bar{L})^2} = \frac{9}{\bar{L}}$  = marginal product of labor in fishing =  $\frac{1}{2}$ . So  $\bar{L} = 18$ ,  $\sum L^i = 180$ . Then output of fish = 100, of wine = 1000 - 180 = 820.

### (b) Welfare Analysis

No. The allocation is not Pareto efficient. The First Fundamental Theorem of Welfare Economics does not apply because there is an external effect in production. The statement of production relations says that the output of fishing firm  $j$  depends on the labor input of  $j$  and the labor input of all other firms. The interaction with other firms is an externality in production, implying that the First Fundamental Theorem of Welfare Economics does not correctly apply to this setting.

The Pareto preferable allocation is to reallocate labor from fishing to wine so that  $\sum_{i=1}^{10} L^i = 100$  but there is no reduction in fish harvest so that it remains true that  $\sum_{i=1}^{10} x^i = 100$ . The additional 100 labor (in the myopic case) or 80 labor (in the sophisticated case) reallocated to wine produces an additional 100 (or 80) wine, to be allocated as an additional 0.1 (or 0.08) wine to each household. Since the indifference curves are flat, it is not possible precisely to characterize the fish and wine consumption of each household, but utility of the typical household goes from 0.50 (or 0.51) to 0.55. A tax of unity,  $\tau = 1$ , on fish, revenue distributed equally to all households will achieve this allocation. This follows since the fishing firm will arrange

marginal value product of labor =  $(p_x - \tau) \times$  marginal product of labor =  $w = 1$ , so marginal product of labor in fishing = 1, therefore  $\sum_{i=1}^{10} L^i = 100$ .

Alternatively, a tax of unity,  $\tau = 1$ , on labor employed in fishing will have the same effect since  $w + \tau = 2 =$  marginal value product of labor in fishing =  $p_x \times$  marginal product of labor in fishing, implies marginal product of labor in fishing = 1, and hence  $\sum_{i=1}^{10} L^i = 100$ .

Though these tax arrangements will support the efficient allocation, we may note that the efficient allocation is not uniquely supported by the tax. Inefficient allocations with  $\sum_{i=1}^{10} L^i < 100$  may be supported by the tax/transfer regime as well.