Problem Set 1

It's OK to work together on problem sets.

The passage below is taken verbatim from the UCSD Economics Micro Qual, June 2007. Hints and directions for you follow the statement of the problem.

On the island of Vinopesce there are two perfectly divisible products: wine, y, and fish, x. The only factor of production is labor, L. Maximum possible fish catch for the whole island is 100 fish. This is a static equilibrium problem: there are no conservation issues. There are ten perfectly competitive fishing firms, denoted j = 1, 2, ..., 10. Labor employed by firm j is denoted by L^j and by firm i (typically a dummy index) is L^i. All firms have the same technology

$$x^j = L^j, \text{ when } \sum_{i=1}^{10} L^i \leq 100$$

$$x^j = 100\frac{\mathrm{L^j}}{\sum_{\mathrm{i=1}}^{10} \mathrm{L^i}}, \text{ when } \sum_{i=1}^{10} L^i > 100$$

Wine is produced under constant returns by many firms k, with the technology, $y^k = L^k$. There are 1000 laborers on Vinopesce, one per household, each endowed with one unit of (divisible) labor. All households have the same utility function

$$u^h(x^h, y^h) = x^h + .5y^h$$

where x^h denotes h's fish consumption, and y^h denotes h's wine consumption. Leisure is not valued. Households sell their labor at the competitive wage rate w. Set $p_y = 1$. That is, wine is the numeraire with price unity. Find the following quantities determined in competitive equilibrium, and explain how you derive them:

$$w = \text{competitive wage rate of labor}$$

$$\sum_{i=1}^{10} x^i = \text{total fish harvest}$$

$$\sum_{i=1}^{10} L^i = \text{total labor employed in fishing}$$

$$p_x = \text{price of fish}$$

total wine output

The First Fundamental Theorem of Welfare Economics says that a competitive equilibrium allocation is Pareto efficient. Is that true of the the competitive equilibrium allocation you derived above?

If yes: explain fully, including the successful tradeoffs, marginal equivalences, etc.

If no: Find a Pareto preferable allocation and explain why the First Fundamental Theorem of Welfare Economics does not apply to this case. Find a tax/subsidy or Lindahl pricing scheme that will achieve Pareto efficient allocation. Explain.

Assignment: Answer the exam question.

Hints: You should be able to answer almost all of the question using the first order techniques in Starr's General Equilibrium Theory, Second Edition draft, Section 1.4 (the 2 × 2 × 2 model), particularly section 1.4.4. The correct answer is an interior solution (there is positive output of both wine and fish) so the marginal value product of labor is equated between the two industries. In order to answer the Pareto efficiency question, draw a production possiblity set and see where the equilibrium output lies. We haven't yet studied tax/subsidy Lindahl pricing in class, so if it's not obvious, don't bother finding a tax/subsidy plan (if it is indeed needed).