Part F

The scope of markets

The models above can be interpreted to treat allocation over time and under uncertainty. In order to do so the space of commodities traded needs to be interpreted to include intertemporal trade and trade in insurance or event-contingent goods. These are the notions of futures markets and contingent commodity markets. The concept of complete markets available over time and uncertainty is sometimes described as “a full set of Arrow-Debreu futures markets.”

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Time and uncertainty: Futures markets

15.1 Introduction

We have already demonstrated the existence and efficiency of general equilibrium in an economy of N goods with active markets for trading them. But what are these N goods? The answer is that they could be anything. This generality reflects the distinctive power of mathematical modeling. The model and its interpretation are separate. We have a mathematical model that provides a general family of results based on mathematical relations among the variables. How we label the variables and interpret the results is now up to us. The model could apply to trading mineral samples at annual meetings of an amateur gemologists society. It can apply to the trading and production of a small closed economy. It can apply to trading and production of an entire world economy. In each case, of course, it applies only if the assumptions of the model are fulfilled. What we know in each instance is that if the assumptions of the model are fulfilled then the conclusions follow: There will be market clearing prices that lead to a Pareto efficient allocation. This is true whether the prices and allocations are for rock samples, the goods available in a small economy, or those available throughout the world. We have left until now a more complete discussion of the range of goods to be allocated by the market mechanism.

The simplest economic models take no explicit account of time. Thus, the model of Chapters 3–14 covers a simple one-period model where all allocation is at a single date. Equivalently, it covers a static, steady-state economy with no intertemporal trade.

Is the general equilibrium model timeless then? Does it have nothing to say about allocation over time? On the contrary, it has a great deal to say about time, allocation over time, and the institutions required for a market economy to achieve efficient intertemporal allocation. It says simply:

> Make the markets for goods over time look just like those in the general equilibrium model and the same formal results will follow. You'll be able to establish an intertemporal equilibrium and intertemporally efficient allocation. All that remains is to interpret what economic institutions it requires for intertemporal goods allocation to look like the general equilibrium model.

That's actually quite a tall order–one that we undertake in the next section.

The simplest economic models take no explicit account of uncertainty. The general equilibrium model covers a simple economic model where all allocation is in a given certain environment. Is the general equilibrium model then without uncertainty? Does it have nothing to say about allocation under uncertainty? On the contrary, it has a great deal to say about uncertainty, allocation over uncertain events, and the institutions required for a market economy to achieve efficient allocation of goods and risk under uncertainty. It says simply:

> Make the markets for goods under uncertainty look just like those in the general equilibrium model and the same formal results will follow. You'll be able to establish an equilibrium for goods across uncertain events and an efficient allocation of risk bearing. All that remains is to interpret what economic institutions it requires for goods allocation under uncertainty to look like suitable goods in the general equilibrium model.

This too is quite a tall order, which we undertake in Section 15.3.

We can outline the character of the economic model's requirements on the space of commodities and firm and household relations to them.

For the market:

- All economically significant scarce resources are traded in the market; goods distinct from one another in production or consumption are distinct coordinates in N-dimensional commodity space.
- There is a single market date at which all supplies and demands are expressed and equated. Budget constraints and firm profits are expressed effective with this date.

For the firm:

- There is a single scalar maximand, profit.
- All economically relevant production possibilities are fully expressed in the firm technology set.

For the household:

- There is a single maximand, $\succeq_i$ or, equivalently, the scalar $u^i(\cdot)$.
- There is a single scalar budget constraint.

For the economy:

- Firm profits are distributed to households. Walras' Law holds.

Our task now is to see how a model of allocation over time and uncertainty can fulfill this outline.

15.2 Time: Futures markets

We can now reinterpret the model above as a model of allocation and economic activity over time. The way we do that is to reinterpret the concept of commodity. Otherwise identical goods deliverable at different dates are to be different commodities. Since firms and households will make their allocation decisions about commodities, they are also making intertemporal allocation decisions.

The idea of a commodity is a primitive concept in the model developed above. The definition of a commodity is implicit in how the notion of commodity enters the model. Two goods are different commodities if they enter separately in the production or consumption decisions of households and firms. If they require different resources to produce them or differ in their consumption desirability, then they are different goods.

In a timeless model with differing geographic locations a commodity is defined:

y what it is (its description), and
y where it is available (its location).

The same good available in two different locations represents two different commodities. After all, a New York driver is not interested in gasoline available in California, and it is a resource-using process (transportation) to convert a gallon of California gasoline to a gallon of New York gasoline. Hence, for the purposes of the model developed in Chapters 3–14, it is perfectly reasonable to interpret deliverable location as a defining characteristic of a commodity. In a one-period model or a stationary equilibrium model then, we distinguish commodities by their delivery location. The model would then be perfectly consistent with differing equilibrium prices of otherwise identical orange juice deliverable in Florida or in Alaska.

Can we apply this same notion to goods separated by time rather than by distance? There are many examples in actual economies of goods distinguished by delivery date. The most prominent is the organized futures markets such as the Chicago Board of Trade, Chicago Mercantile Exchange, or the New York Commodities Exchange. In these markets there is active trade in grains, metals, fibers, petroleum, and foods, specified by description, quality, place of delivery, and by date of delivery. Contracts for goods otherwise identical in description and location may trade at prices differing substantially by date of delivery. It is a resource-using production activity (storage) to convert goods deliverable at one date into goods deliverable at a succeeding date. Goods deliverable in the distant future may trade at prices far different from those in the present. Prices for future delivery may be lower than current delivery (spot) prices, reflecting the anticipated availability of additional new harvests or other supplies becoming available. Alternatively, current prices for future delivery may be higher, reflecting storage costs. Prices payable currently for future delivery may be lower than for spot delivery, reflecting time discounting. That is, prices are in the nature of present discounted values, discounted from the delivery date back to the market date.

We can take this notion of futures prices and discuss our general equilibrium model where there is a full set of futures markets. A commodity is characterized

y what it is (its description),
y where it is available (its location), and
y when it is available (its date).

There are actively traded goods for all dates: If a good will be available at a particular date in the future, futures contracts for the good deliverable at that date are traded in the market at the market date. The formal mathematical model of production and consumption remains completely unchanged by this change in interpretation. However, to understand the implications of this augmented model of futures markets requires some economic interpretation.

Let's start with N, the number of commodities. We take N to be finite. The number N includes as a separate count every good, at every location where it is deliverable, and at every date at which it is deliverable. N is clearly a large finite number. Assuming N is finite amounts to assuming that there is a finite number of locations at which goods can be delivered and that there is no significant spatial difference within each location. More importantly, assuming N is finite means that in terms of economic time there is an ending date, and so we are using a finite horizon model. The finite horizon may be very far away (e.g., 10,000 years is a finite number), but this artificial construct is unfortunately mathematically essential. We could interpret this as indicating a true determinate predictable end to economic activity. Alternatively, we could interpret the finite horizon as a time so distant that prospects beyond the horizon can have no effect on supply and demand on futures markets meeting in the present.

The trickiest issue involves interpreting the prices of goods, $p \in P$,$p=(p_1, p_2, \ldots, p_N)$. There is only a single meeting of the market. The market mechanism – personified as a Walrasian auctioneer–simultaneously balances supply and demand for all dated goods. Each household has only a single budget constraint, representing receipts and expenditures at all dates from the present to the finite horizon. Firms have only a single calculation of profit, representing the net return on receipts for outputs and expenditures for inputs over all dates from the present to the finite horizon. All receipts and expenditures for spot (current) goods and future deliveries are evaluated at the single market date. Hence we can interpret p_i, the price of commodity i (where the description of commodity i includes i's delivery date), as a present discounted value of commodity i discounted from the delivery date to the market date. This model is usually described as including "a full set of futures markets," that is, markets currently available for all goods at all future dates.

The convention on payment for futures contracts bought and sold is institutionally a bit different here from those in operation in actual economies. Our model requires payment at the market date, far in advance of delivery. In contrast, at the Chicago Board of Trade, agreements to buy or sell commodities may be undertaken years in advance; full payment is made only at delivery. In the present model, all of the financial elements of economic activity take place at the single market date prior to the rest of economic activity. Costs are incurred, revenues received, accounts debited and credited at the market date, long prior to delivery. This reflects an assumption of full reliability of the agents without possibility of default on the promised deliveries.

How do we interpret the household endowment $r^h \equiv (r_1^h, r_2^h, \ldots, r_N^h)$? The household is endowed with present and future goods. The household typically is endowed with its own labor deliverable in the present and in each of the next several periods, up until the date of its death. In addition the household may own other dated goods. If it owns land, its rights to the use of the land are time dated from the present up until a finite horizon. A similar situation occurs for other real goods with which the household is endowed (we deal with share ownership α^{hj} in a moment).

How can we describe household consumption $x^h \equiv (x_1^h, x_2^h, \ldots, x_N^h)$ in this economy with complete futures markets? Each coordinate in x^h represents dated planned consumption of a particular good. Hence the vector x^h comprises a list at each of the dates in the present and the future of planned consumption at that date. It represents a lifetime consumption plan for household h.

Similarly, firm j's production $y^j \in Y^j$ represents a dated plan for inputs and outputs at a sequence of dates. Thus, seeds, labor, and the use of land in the spring result in a harvest in the fall. Grapes, barrels, and a cellar in 1995 result in good wine in 1996 and excellent wine in 1997. Capital in 1995, 1996, ... combined with labor and intermediate inputs create output in 1995, 1996, ... The set Y^j then represents an array of technically possible plans of mixing dated inputs to produce dated outputs from the present through the finite horizon for firm j. Among the production possibilities, of course, is $0 \in Y^j$, the possibility of not operating firm j actively at all.

Input and output prices are discounted values, discounted to the market date. At prevailing prices $p \in P$, firm j's profit is

$$\pi^j(p) = \max_{y \in Y^j} p \cdot y = p \cdot S^j(p).$$

That is, $\pi^j(p)$ is the sum evaluated at the market date, over all dates from the present through the time horizon of the (present discounted) value of outputs less the (present discounted) value of inputs. Firm j's supply behavior $S^j(p)$ is then characterized as choosing a production plan in the present and for all future dates to maximize the present discounted value of the flows of outputs less inputs of the firm. The profit $\pi^j(p)$ is the value of firm profits discounted to the market date or, equivalently, a present discounted value of the flow of firm profits. Maximizing firm (discounted) profit and maximizing firm (stock market) value are identical.

In actual economies, markets meet at each date, and receipts and expenditures take place at each date. In this model, receipts and expenditures take place only at the market date though delivery of goods takes place throughout time. The presence of the complete futures markets allows all of the receipts and expenditures of the firm representing current and future deliveries to be collapsed into a single number representing the present discounted value of the firm's profits. Hence $\pi^j(p)$ represents the (stock market) value of the firm. The presence of the complete futures market eliminates the distinction between the value of the firm and it's stream of profits by collapsing the future into the single market date. The complete futures market eliminates the stock-flow distinction between income and wealth.

The preferences of household h, $\succeq_h$, represent preferences on time-dated streams of consumption from the present through the future until the horizon. The preferences $\succeq_h$ include h's attitude toward consumption timing (impatience) as well as desires for variety and consistency in consumption over time. Household h's preferences into the distant future are taken to be fully predictable (since this is a subjective certainty model).

The value of endowment and goods prices are discounted values, discounted to the market date. As before, household h's income is characterized as $M^h(p) = p \cdot r^h + \sum_{j \in F} \alpha^{hj} \pi^j(p)$. Since $M^h(p)$ includes pricing for all goods and profits into the future, it can be interpreted as a measure of wealth (a stock) rather than income (a flow). In the presence of the full set of futures markets the stock/flow distinction becomes irrelevant. Household consumption behavior is characterized as before. Household h chooses $x^h \in X^h$ to optimize $\succeq_h$ subject to $p \cdot x^h \leq M^h(p)$. That is, h chooses a consumption plan for the present through the horizon to optimize a planned program of consumption evaluated by h's preferences for consumption across goods and time. It does so subject to the budget constraint that the present discounted value of the consumption plan is bounded above by the present discounted value of endowment plus the value of firm ownership (this latter equals the discounted value of the flow of outputs less inputs from the firms).

Market equilibrium is characterized as prices $p \in P$, a price for each dated good representing a present discounted value, so that all markets clear. That is, for each good at each date the futures market demand for the dated good is equated to the futures market supply with the possibility of free goods in oversupply.

Here is what the economic activity looks like in this model. The market takes place at a time prior to all economic activity. Prices are quoted for all goods at all current and future dates up to a finite horizon. Prices of future goods may be conceived as present values discounted to the market date. At those prices firms formulate a production/supply plan that maximizes the value of the firm. This is equivalent to maximizing the discounted value of the dated stream of firm profits earned through sales and purchases deliverable at the succession of dates. Household budgets are formulated as the value of endowment (equivalently, the discounted value of the dated stream of endowed goods) plus the value of firm ownership, both evaluated at the market date. The household then chooses a consumption plan to satisfy preferences subject to the budget. The value of the consumption plan (discounted value of the dated stream of goods consumption) is constrained by the budget. Equilibrium is characterized as a price vector for the array of goods that equates supply and demand for all dated goods. The household comes to market with a dated endowment stream and delivers the endowment to the market. It leaves the market with contracts for a consumption plan for the present through the horizon. That is the only meeting of the market. Because markets are complete and there is no uncertainty, reopening the market would serve no function–there would be no transactions. The balance of economic activity from the market date to the horizon consists in fulfillment of the contracts undertaken on the futures market. As usual, equilibrium is Pareto efficient. There is no reallocation of goods or factors across firms, households, or over time that would create a Pareto-improving reallocation. Household well-being here is judged not at a single point in time but rather over the lifetime up to the horizon, according to household intertemporal preferences.

The notion of a household becomes a bit more complex in this setting since the household is active in the market at the market date and the model extends through a finite horizon. How can we deal with the unborn? The model is of course silent on this, but it gives scope for interpretation. All households are represented in the market. How can we interpret the unborn? Someone who is unborn at date 1 merely means that they have no endowment dated 1 and prefer to avoid consumption until some later date, b, their birthdate. Who represents their preferences at the market? Although the model tells us nothing, it is clear that for the allocation to be an equilibrium and efficient, they will require representation. An alternative interpretation is that though there are individuals unborn at the market date, there are no unborn households. Unborn individuals' interests are represented by their parents or other ancestors. These are admittedly unsatisfactory replies.

The futures markets here perform the functions both of goods markets and of capital markets. Thus the household budget constraint is in the nature of a lifetime budget constraint. The present discounted value of the household lifetime consumption plan is bounded by household wealth, the present discounted value of endowment plus firm ownership (the household's share of the present discounted value of firm profits). In a model without futures markets this value would be comparable to the value of wealth plus the discounted value of future income streams. The complete futures markets eliminate the distinction between income and wealth. The complete futures markets imply a perfect capital market: There is no effective borrowing constraint on current consumption other than eventual ability to repay. There is no effective constraint on firm investment other than the eventual profitability of the business undertaken. All trade takes place prior to consumption or production. Consumption in one period can be financed by delivery of endowment dated before the consumption takes place (corresponding to saving by the household in a model without futures markets) or after the consumption (corresponding to borrowing). Firms finance their purchase of inputs through the sale of outputs. The outputs may be dated later than the inputs. That is precisely the function of capital markets–the forward sale of outputs finances the prior acquisition of productive inputs.

15.2.1 A sequence economy

The futures market model can seem a bit daunting. It requires so many markets to be available and active at the market date. And it requires that all market activity stop after the single active market date. It seems painfully unrealistic.

There is an alternative, one that carries most of the same structure without the requirement of so many active markets at a single date and that allows markets to reopen. That is the model of a sequence economy, which is equivalent to the futures market model.

The sequence economy is characterized in the following way: At each date there are spot markets for active trade in goods deliverable at that date. There are financial markets in debt instruments–borrowing and lending into the future. Firms and households have perfect foresight concerning the prices prevailing in the future. At each date, firms and households buy and sell spot goods. They face a budget constraint at each date: Sales of goods and debt (borrowing) must finance purchases. To the extent that their purchases on the current market exceed their receipts, they borrow. To the extent that their receipts exceed their expenditures, they lend. At the finite horizon they must fulfill a lifetime budget constraint: No one can be a net debtor at the end of the finite horizon. Equilibrium occurs when all markets clear at each date, both spot good markets and the debt markets. With perfect foresight regarding future spot prices, it is easy to show that the sequence economy model is equivalent to the complete futures market model. Foreseen spot market prices (correctly foreseen to be equivalent to the futures market prices) replace futures prices. Debt markets replace futures markets in redistributing purchasing power over time. Essentially, a simple reinterpretation of the futures market model with the addition of debt instruments allows us to model intertemporal allocation without explicitly resorting to futures markets. This certainly appears more realistic. Of course, it relies on the unrealistic assumption of perfect foresight on spot market prices to replace the unrealistic model of complete markets. The sequence economy model with complete debt markets corresponds to the concept of a perfect capital market.

15.3 Uncertainty–Arrow-Debreu contingent commodity markets

Time is not the only complication in designating the commodities of economic activity. There is also uncertainty. Economically important events that we cannot clearly foresee include the weather, our health, and technical change. It is formally possible fully to take account of uncertainty again through a very clever reinterpretation of the model we already have in place.

We have heretofore defined a commodity by description, location, and date. We now go a step further. Uncertainty means that we don't know what's going to happen in the future. But we do know what might happen. Assume that we can make an exhaustive list of all the uncertain events that might take place in the future. We describe this array of possible events by an event tree (see Figure 15.1). At each date there is assumed to be a finite list of events that describes the condition of the economy in terms of all the economically relevant uncertain events that may occur. The path of events in the economy is framed as transit down one of the branches of the event tree. A state of the world will be defined by the current condition (in terms of uncertain events) of the economy and the history of past realizations of uncertain events that leads to it.

In Section 15.2, we reinterpreted our basic model to accommodate time by defining the idea of a commodity to include specification of a delivery date. We now perform the same reinterpretation to accommodate uncertainty by defining a commodity to include specification of a state of the world. A commodity is now characterized

by what it is (its description),
by where it is available (its location),
by when it is available (its date), and
by it's state of the world (the uncertain event in which it is deliverable).

The number of commodities N has grown again. Again we take N to be finite. That means we are assuming that the number of possible uncertain events is finite at every date (in addition to the previous assumption that the number of time periods is finite).

What is a commodity in this setting? It's not really something you can use or consume. Rather, it is a promise of delivery of a particular good or service at a particular date if an uncertain event actually occurs. The term for that is a contingent commodity. This sounds a bit bizarre, but we have all experienced contingent commodities. An HMO (health maintenance organization) medical plan is a contingent commodity (or a bundle of contingent commodities). It is a contingent commodity providing medical care in the uncertain event that you are ill or injured. An auto club membership is also

Figure15.1

Fig. 15.1. Uncertain states of the world: An event tree.

a contingent commodity. It provides towing and emergency repair service in the uncertain event that your car malfunctions. An insurance contract is a closely related concept. Insurance usually provides a payment of money in case a specified uncertain event occurs–that's not precisely the same as a contingent commodity, but it's similar if the payment is chosen to cover the cost of a particular purchase you want to make in the event. We discuss this further in Section 15.4.

The price of good i will not generally be the price of a definite consumption. It is the price of a contingent commodity, the price of a specific good deliverable if a specified event occurs.

What is the meaning of y^j, firm j's production plan, in this setting? Prior to the start of economic activity, j's management considers the production possibilities along each branch of the event tree. For a farming enterprise, the production possibilities might look something like this: Inputs of land, labor, and seed in the spring produce an uncertain output. There are three events to deal with: drought, normal rain, and flood. In each event there will be an output, but the quantity will differ by the event. Thus, the production possibilities of j are well specified though uncertain only because of the uncertainty of the weather. Firm j then consults its technology Y^j and the prevailing prices of contingent inputs and outputs. It will choose a plan y^j that specifies the inputs it needs and the outputs it plans to produce in each event and date. It makes a plan for each branch of the event tree–actual events will take it only along one branch of the tree. It may buy inputs and sell outputs along each branch of the tree, wherever the currently prevailing prices make this purchase and sale of contingent commodities profitable. Consequently, most of its planning will never be implemented. Most of the contingent commodities it buys and sells will not be delivered, since the events in which they are deliverable may not take place. The firm needs no attitude toward risk-taking or risk aversion. The firm's production plan is chosen to maximize the value of $p \cdot y$ for y in Y^j at contingent commodity prices that are known with certainty at the market date (prior to the rest of economic activity). In order to make this choice of profit-maximizing contingent production plan, the firm does not need a probability judgement to forecast which states are more likely nor does it need an attitude toward risk. Its production opportunities are fully specified by Y^j; the profitability of any plan is fully implied by p. Implicit in this formulation is the concept that the firm's supply decisions are default free. Even if the firm (or its managers) believes the probability of an event occurring to be nil, it will sell output in that event only to the extent that it purchases contingent inputs that will allow production of the projected output in the unlikely situation that the event actually takes place. In equilibrium, households' risk aversion and probability judgements will be emobodied in the contingent commodity prices.

At prevailing prices $p \in P$, firm j's profit is

$$\pi^j(p) = \max_{y \in Y^j} p \cdot y = p \cdot S^j(p).$$

That is, $\pi^j(p)$ is the sum evaluated at the market date, over all dates and events of the (present discounted) value of contingent outputs less the (present discounted) value of contingent inputs. Firm j's supply behavior $S^j(p)$ is then characterized as choosing a production plan in the present and for all future uncertain events to maximize the present discounted value of the flows of contingent outputs less contingent inputs of the firm. Maximizing firm profit and maximizing firm (stock market) value are identical.

A household h's endowment vector r^h is an N-dimensional vector listing the endowed contingent commodities of the household: 24 hours a day of labor/leisure in the event the household is alive and well, 0 in the event the household is dead, and so forth. As before, household h's income is characterized as $M^h(p) = p \cdot r^h + \sum_{j \in F} \alpha^{hj} \pi^j(p)$. The household sells all its endowment r^h. The endowment consists of contingent commodities, most of which will never actually be delivered (since their events may not take place). Nevertheless, the full endowment of contingent commodities is sold forward and the proceeds enter h's budget.

Household h's consumption vector x^h represents a state-contingent dated list of projected consumptions. Each coordinate in x^h represents a dated contingent consumption of a particular good in its specified state of the world. The vector x^h is a list, at each date and state, of planned consumption at that date/state pair. It represents a lifetime event-contingent consumption plan for household h. The preferences of household h, $\succeq_h$, represent preferences on time-dated, state-contingent commodities from the present through the future until the horizon. Household h considers the prospect of each possible mix of contingent commodities, and $\succeq_h$ represents h's preferences among them. Since the contingent commodities are not precisely consumptions, it is not precisely accurate to say that $\succeq_h$ represents h's consumption preferences. Rather, $\succeq_h$ represents h's preferences among contingent commodity consumption programs, preferences that reflect the result of h's consumption preferences on actual goods when delivered, h's personal judgments on the likelihood that the individual uncertain events will actually take place, and h's attitude toward risk (unpredictable variation in consumption). Vector x^h represents a portfolio of risky assets. The preference ordering $\succeq_h$ then represents h's preferences among those portfolios.

One way to think of the formulation of $\succeq_h$ is to regard the preference ordering on contingent commodities as representing an expected utility. This is the most easily interpretable formulation. Nevertheless, assuming expected utility optimizing behavior is not necessary to pursue the model. Any transitive continuous preference ordering on portfolios of contingent commodities will do the job. The assumption of convex preferences, C.VI or C.VII, will typically be maintained; that implies risk-averse behavior. A risk lover will concentrate his portfolio on consumption deliverable in a single event–he doesn't want to hedge his bets. However, convex preferences on the portfolio imply that given the choice of two equally desirable portfolios, each with its payoff concentrated in a different single event, the midpoint of the two portfolios will be preferred to either extreme. The midpoint represents hedging–not putting all your eggs in one basket. That's risk aversion.

In this model of contingent commodities, household h's demand behavior is characterized just as before. Household h chooses $x^h \in X^h$ to optimize $\succeq_h$ subject to $p \cdot x^h \leq M^h(p)$. That is, h chooses a state-contingent dated consumption plan for the present through the horizon to optimize the consumption plan evaluated by h's portfolio preferences for contingent commodities subject to a wealth constraint. A portfolio will imply a dated consumption plan across time along each branch of the event tree. Risk takes the form of possible variation in consumption across events. The way for the household to assure a steady consumption is to choose contingent commodities that deliver the same consumption plan independent of events. Alternatively, at the market date, the household can adjust its contingent consumption plans to vary with the market's price differentials. Market prices for each commodity will reflect the differing scarcities of goods across events, household state-varying tastes for the goods (desirability of the good an umbrella in the rain will differ from that of an umbrella in dry weather), household attitudes toward risk, and household probability judgments on the likelihood of the states of the world. Household h chooses its optimal portfolio subject to the budget constraint. The budget constraint says that the value of the portfolio of contingent commodities chosen is bounded by the value of the contingent commodity endowment plus the value of the household endowment of firm shares (whose value is also determined on the contingent commodity market).

Equilibrium in this contingent commodity economy occurs just as in the certainty economy with futures markets. The market prices all of the contingent commodities. Supplies and demands are announced by firms and consumers. Prices adjust until supply equals demand. Households come to the market with their endowed contingent commodities and sell the endowment. They acquire a portfolio of contingent commodities that represents their most desirable portfolio subject to budget constraint. Payment takes place at the market date. The profits of firms, the value of household endowments, and the value of household budget constraints and of household consumption plans are all computed in terms of the prices of these plans at the contingent commodity prices. The household budget constraint applies at the single market date. All contingent commodities are bought, sold, and paid for at the market date. Payment is made for the contingent commodity contract, not for actual delivery (which may never take place).

Since most of the possible states of the world do not take place, most contingent commodity contracts expire without being executed by delivery. In the absence of any learning or change in subjective probabilities or tastes, there is no need for markets to reopen. If they did reopen there would be no active trade on them. Once the equilibrium is established, remaining economic activity in the economy consists merely in the execution of the contracted plans. At each date households and firms discover the state of the world. They discard as worthless all of their contracts for contingent commodities deliverable in other states at that date and contracts for future delivery in branches of the event tree that they now know will not take place. They then deliver and take delivery on the contracts for the date-state pair that pertains. The balance of economic activity through the horizon consists of fulfilling their previously contracted plans.

The equilibrium allocation of risky assets is Pareto efficient relative to $\succeq_h$, that is, relative to household preferences on contingent commodity portfolios. Given the endowments r^h and available technologies Y^j, there is no attainable reallocation of inputs to firms j or of contingent commodity outputs to households h that would move some household h higher in its ranking of portfolios, $\succeq_h$, without moving some other household h' lower in its ranking of portfolios, $\succeq_{h'}$. This means that the allocation of risk bearing among households is Pareto efficient. There is no rearrangement of the risky assets, the contingent commodities, among households that would be Pareto improving in terms of household portfolio preferences.

We should recognize as well what Pareto efficiency of the allocation of contingent commodities does not mean. The concept of efficiency here takes the probability judgments of households as both exogenous and given. It is perfectly consistent with our concept of efficiency that we could improve the allocation of goods actually delivered by improving household foresight of the future. All the market does is to efficiently implement the allocation of contingent commodities subject to prevailing expectations. Efficiency of the allocation of contingent commodities does not assure us that there will be no regrets. After the state of the world is revealed, many agents will discover that their expectations were mistaken and they will wish that they had arranged their portfolios differently. Indeed, their mistaken expectations may cause a real misallocation of resources. Widely held expectations may raise contingent commodity prices for goods deliverable in an expected event. Those high prices for the expected event then may lead to input reallocations that skew output toward the expected event away from other events. For example, if most households expect flooding then market prices of output deliverable in the event of flooding will be higher (than they would otherwise be) as well. Farms wishing to produce output deliverable in that event to take advantage of the high prices will reallocate planting to forms that will deliver in the event of flooding (e.g., planting on high ground at additional expense of resources). These additional resources will turn out to have been wasted if the flooding does not take place. The markets efficiently allocate resources, consumption, and risk for a given state of expectations of the future. They provide no substitute for foresight.

15.4 Uncertainty–Arrow securities markets

Contingent commodity markets provide in equilibrium for an efficient allocation of risk bearing. Within each date-event pair, they provide an efficient allocation of goods. They do so at potentially great cost either to realism or to the operating costs of the markets, for this model requires a great many markets to be active at the market date, and none to be active thereafter. This model requires that each good be traded at the market date before the start of economic activity in a multitude of different contracts. There will be a different contingent commodity for each good, date, and event combination. Since each node on the event tree constitutes a different event at the date represented, the proliferation of date-event pairs is immense. And the model requires that each good be traded in a separate contract for each such pair! This is an overwhelming proliferation of contingent commodities! The model calls for many more active markets at the market date than we ever see actually in reality–and it calls for far fewer at most dates in real time than we actually see in market economies. How can we escape this bind? Can we retain the essential elements of this model–market allocation of goods and risk–while moving to greater realism, fewer active markets for risky goods, and more active spot markets?

It is possible to restate the model of the contingent commodity general equilibrium in a way that retains all of the results but lets us significantly reduce the number of markets in active use at each date and allows trade to reopen at each date, adding a touch of realism to the model. We define an Arrow insurance contract in the following way: Suppose there is a "money" or numeraire in which we can describe a payment of generalized purchasing power. For each date-event pair, t, s, the contract $c(t, s)$ pays one unit of purchasing power if event s occurs at date t and nil otherwise. Then, instead of a full set of contingent commodity markets, we can use a mix of insurance contracts and spot markets (markets for actual goods deliverable in the current period) to achieve the same allocation as available in the contingent commodity equilibrium. In order to make their portfolio decisions however, households and firms will need (perfect) state-contingent price foresight. They'll need to know what spot prices to expect for each good in each event.

In designating a commodity i, we have not thus far needed to distinguish i by the date or event in which it is deliverable. It is time to do that now. Using a somewhat imprecise notation, let us write $i \in (t, s)$ if good i is deliverable at date t, state s, and of course $i \notin (t, s)$, if not. Now consider the value of h's spending on contingent commodities deliverable in (t, s), $\sum_{i \in (t,s)} p_i x_i^h$. That is the amount at currently prevailing contingent commodity prices that household h spends on the contingent commodity market for goods deliverable at date t, state s. Suppose we then reopen the spot markets for goods in (t, s). Denote the spot price of good $i \in (t, s)$ on the spot market at t as q_i. Finally, let the price of an Arrow insurance contract payable in (t, s) be $\theta_{t,s}$. Let household h buy $S_{t,s}^h$ units of Arrow insurance contract $c(t, s)$, where

$$S_{t,s}^h = \sum_{i \in (t,s)} q_i x_i^h.$$

For $i \in (t, s)$, set $p_i = \theta_{t,s} q_i$. Then the household budget constraint can be restated as $\sum_{t,s} \theta_{t,s} S_{t,s}^h \leq M^h(p) = M^h(\theta, q)$. Here θ and q denote the vectors of $\theta_{t,s'} q_i$. Thus the household budget (and hence the entire household optimization problem) can be restated in terms of the prices of Arrow insurance contracts $\theta_{t,s}$ and the spot prices q_i without any direct reference to the contingent commodity markets or their prices p_i.

A firm's policy in this economy is to formulate its profit-maximizing production plan, just as it did in the full contingent commodity model. The firm needs no attitude toward risk. Like households, it does need to have correct state-contingent price foresight. That is, the firm correctly foresees that if event s occurs at date t, then the price of good i will be p_i. The firm then maximizes its value (the present discounted value of the stream of state-contingent outputs less the cost of inputs it plans) based on its technology and the correctly foreseen state-contingent prices and Arrow securities prices. It announces its planned profits to its shareholders who incorporate the announced values in their budget constraints. In each date-event pair, the firm may have a deficit or surplus of receipts less disbursements attributable to that date-event should it occur. The firm finances its production plan by trading on the Arrow securities markets and distributing profits to shareholders. The value of the firm profits (its stock market valuation entering the owners' budgets) equals the value of its securities sales less its purchases. The demands of price foresight here are significant (and implausible), but so is the reduction in the volume of transactions and corresponding increase in verisimilitude. Indeed, in actual market economies with well-developed financial markets, firm stock market values do indeed enter owners' budget constraints and represent a present discounted value under uncertainty of future profit streams.

What we have just argued is that a family of simple accounting identities can create a formal equivalence between two quite different models. The first (Model I) is the model of the contingent commodity markets:

> The market meets once for all time and a very large number of contingent commodities are traded; most do not result in delivery of actual goods.

The second (Model II) is a model of securities markets for securities (Arrow insurance contracts) payable in abstract purchasing power:

> The securities market meets once; goods markets reopen at each date for spot trade. Most securities do not result in actual payment.

We claim that Models I and II are equivalent. The key to this equivalence is simply that in Model II spot relative prices for goods in each state should be the same as their relative prices in Model I and that the securities positions assumed by traders in Model II be sufficient at the resultant spot market prices to support their consumption plans from Model I.

What can we conclude? We can replace the full set of contingent commodity markets discussed in Section 15.3 with a much smaller number of markets. Instead of a market for each good deliverable in each date and event, we can use a securities market that distributes purchasing power across dates and events. In those events where a firm is profitable or a household has a large endowment, the model replaces the remuneration for those real goods with the value of securities payable in money for the date-event pair. In each date and event, once the event that actually pertains is clear, spot markets for factors of production and for consumption goods open to distribute the actual goods for consumption and factors to use. Instead of maintaining a full set of contingent commodity markets for all goods deliverable in all events, the only goods markets actually in use are those for events that actually take place. There are active securities (or insurance) markets, one for each possible date-event combination. The capital market function of the contingent commodity markets is fulfilled by the securities markets: To finance activity in one date-event from the anticipated proceeds of another, sell securities from the second and spend the proceeds on securities payable in the first.

To demonstrate this equivalence, firms and households need perfect price foresight for each date-event pair in the future. How else will they know the value of securities to buy and sell? At the market date all of the firms and households must know what the spot market prices q_i are going to be. The N commodity markets do not all need to meet, but the economy needs to use the information that they would generate. However, generating the equilibrium prices is a prime responsibility of the markets. We may argue that this is too much foresight for the model to require; how can market prices be known even before the markets meet? Alternatively, we can argue that the requirements of the model are plausible; households may reasonably be expected to have a good forecast of market prices under well-specified events (for example, they would expect agricultural prices to be higher in the event of bad weather than in good). Further, it is not necessary for all agents to foresee all prices. They need only know the value of firms and of the budgets they need in each date-state. These are summaries, not individual prices. Nevertheless, the notion of perfect price foresight is troubling. It is particularly hard to defend in the case of multiple equilibria, where even the Walrasian auctioneer with full information cannot predict which of several possible equilibria will prevail.

15.5 Conclusion–the missing markets

The use of futures markets, contingent commodity markets, and Arrow insurance markets (with perfect date-state price foresight) allows the market mechanism to overcome the confusion generated by time and uncertainty. Markets can work successfully when there are enough of them. We need a sufficient variety of commodity and financial instruments traded in the market to allow the market allocation mechanism to do its job. Unfortunately, this model appears to require many more active markets than are actually in use in real economies. The financial markets of a modern economy, including stock exchanges, futures exchanges, option exchanges, and the (dealer) market for insurance instruments not sold on exchanges, provide an array of markets for intertemporal allocation and exchange of risk that is rich and complex. Nevertheless, they are sparse compared to the array of possible uncertainties and dates facing economic agents.

The message of this family of models is that a rich enough array of active markets can result in a successful allocation over time and uncertainty. Conversely, one source of allocative failures in actual economies is the absence of a sufficiently large array of future and contingent commodities actively traded. A persistent objection to the class of models is that they require far too many active markets–many more than will be found in an actual economy. The reasons for these mismatches between theory and practice are not to be found in the theory; they reflect issues omitted from the model: the costs of operating markets themselves, and the difficulty or cost of verifying the state of the world.

The major results articulated in Chapters 3–14 for an applied economist or policy maker are a restatement of the laissez-faire doctrine: The market will perform allocation decisions and do it right. The discussion in this chapter points out a strength and a weakness in that message. We have demonstrated the power of that formal result by showing that it persists over time and across uncertainty. We have demonstrated its fragility by showing that it requires many more active markets than actual economies contain. When a laissez-faire advocate insists that the market makes the best allocation decisions, he or she is using the fundamental theorems of welfare economics. The advocate doesn't typically stop to qualify such claims for the market by noting that the proposed economy lacks sufficient insurance markets fully to handle uncertainty or capital markets perfect enough fully to deal with intertemporal allocation.[1]

15.6 Bibliographic note

The brilliantly simple notion of dated commodities first appears in Hicks (1939). The notion of contingent commodities and of Arrow insurance contracts appears in Arrow (1953, 1964) and is well expounded in Debreu (1959).

[1] The bridge between theory and application requires luck and interpretation. All theories in the sciences are abstract, but they give predictions about concrete results. That's true in physics and chemistry as well as in economics. No theory perfectly fits application. The theory is a guide to application. It's a judgment call when the omissions of the theory are sufficiently great and relevant to cause a failure in application.

Exercises

15.1 Consider the economy with a finite time horizon and a nonrenewable natural resource (e.g., coal or oil). In each of the following cases describe the process of decision making with regard to use of the nonrenewable resource and state whether the allocation may be expected to be Pareto efficient. Will the economy run out of coal or oil because of excessively rapid use? Why or why not? Explain.

CASE 1 A full set of futures markets. There are active futures markets for the resource and its products available for delivery at all present and future dates.

CASE 2 No futures markets, perfect foresight, and perfect capital markets. There are no active futures markets but there is perfect price foresight regarding the resource, its outputs, and all other goods. All agents have access to a perfect capital market that allows them to borrow and lend, and spend and save, at common equilibrium interest rates, subject only to a lifetime budget constraint.

CASE 3 No futures markets, no active capital markets, perfect price foresight. Saving and investment decisions are taken but they are autarkic– households have no access to a market for borrowing and lending.

15.2 Consider an economy in general equilibrium with a full set of Arrow-Debreu contingent commodity markets. Explain how the economy deals with medical insurance. How does it work? Is medical insurance just another contingent commodity? Is there a moral hazard problem (overspending when the insured event occurs since insurance will cover the bill)? Will every household be insured for every illness or injury?

15.3 Consider a firm planning to start operations in an intertemporal certainty economy with a full set of futures contracts. There are profitable opportunities to produce widgets for supply at $t+2$; this production requires inputs at t. The firm is inactive prior to t. How does the firm finance its production plan?

15.4 Consider a firm deciding the production and sale of output in an uncertainty economy with a full set of Arrow-Debreu contingent commodity contracts. Let there be two dates, 0 and 1. There is one state in date 0 and three states in date 1 (denoted 1.1, 1.2, and 1.3).

(i) Describe the decision making of the firm.

(ii) Explain the trade-off's as the firm chooses among producing for each of the alternative contingencies 1.1, 1.2, and 1.3.

(iii) If production is intertemporal (requiring inputs at one date to produce output at a succeeding date), explain how it is financed.

(iv) Does risk aversion of the firm's management or owners enter the production decision? Explain.

15.5 Consider education as a private investment good. Explain the following observations:

(a) In the Arrow-Debreu Walrasian model with a full set of futures markets, efficient allocation of resources does not require government provision of education. The market will provide and distribute education in a Pareto efficient fashion.

(b) In actual economies, market imperfections may prevent private markets from financing efficient levels of education. This may create a role for nonmarket provision or explicit subsidy.