Part D
Welfare economics

Ever since Adam Smith's evocation of an invisible hand, market equilibrium has been supposed not only to clear markets but also to achieve an efficient allocation of resources. This view is embodied below in Chapter 12 in a definition and two major results. We define a very general efficiency concept, Pareto efficiency. We then state and prove the two major results relating market equilibrium to efficient allocation, which are the two most important results in welfare economics.

The First Fundamental Theorem of Welfare Economics agrees with Adam Smith: A market equilibrium allocation is Pareto efficient. This result can be demonstrated in a surprisingly elementary fashion. It requires very little mathematical structure and it does not require any assumption of convexity. If, despite nonconvexity, the economy has a market equilibrium, that equilibrium allocation is Pareto efficient.

The Second Fundamental Theorem of Welfare Economics requires more mathematical structure. It is a more surprising and deeper result. It says–assuming convexity of tastes and technology–that any efficient allocation can be supported as a competitive equilibrium. Find an efficient allocation. Then there are prices and a distribution of resource endowments of goods and share ownership that will allow the efficient allocation to be an equilibrium allocation at those prices and endowments. Market allocation is compatible with any efficient allocation subject to a redistribution of income.

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Pareto efficiency and competitive equilibrium

12.1 Pareto efficiency

The purpose of economic activity is to allocate scarce resources to promote the welfare of households in their consumption of goods and services. There is a very large number of possible allocations of resources (typically an uncountable infinity), but most of them are wasteful–we can do better. Some wasteful allocations are those that do not make effective use of productive resources (corresponding to points inside the production frontier in the Robinson Crusoe economy). An alternative form of inefficiency occurs in allocations that allocate the mix of outputs among consumers without equating marginal rates of substitution, leaving room for improvement in the mix of consumption across households (wasteful points corresponding to those off the locus of tangencies in the Edgeworth box).

Economic theory does not give us precise guidance as to the desirable distribution of income and wealth across households. The theory is agnostic on the distribution of income between Smith and Jones and between Rockefeller and Micawber. We are led then to posit a criterion of nonwastefulness as a standard for the effective utilization of scarce resources, while avoiding the moral question of the desirable distribution of income. The nonwastefulness criterion is Pareto efficiency, and it is fundamentally a simple idea. A (Pareto) improvement in allocation is a reallocation that increases some household's utility (moves higher in the preference quasi-ordering) while reducing no household's utility. An allocation is Pareto efficient if there is no further room among attainable allocations for (Pareto) improvement.

To analyze this concept more fully we start with the definitions needed to formalize these concepts.

Definition An allocation x^i, $i{\in}H$, is attainable if $x^i \in X^i$, $i{\in}H$ and there is

$y^j \in Y^j$, $j \in F$, so that $0 \leq \sum_{i\in H} x^i \leq \sum_{j\in F} y^j + \sum_{i\in H} r^i$. (The inequalities hold coordinatewise.)

Definition Consider two assignments of bundles to consumers, v^i, w^i, $i \in H$. v^i is said to be Pareto superior to w^i if for each $i \in H$, $v^i \succeq_i w^i$, and for some $h \in H$, $v^h \succ_h w^h$.

Definition An attainable assignment of bundles to consumers, w^i, $i \in H$, is said to be Pareto efficient (or Pareto optimal) if there is no other attainable assignment v^i so that v^i is Pareto superior to w^i.

Definition $\{p^0, x^{0i}, y^{0j}\}$, $p^0 \in \mathbf{R}^N_+$, $i \in H$, $j \in F$, $x^{0i} \in \mathbf{R}^N$, $y^{0j} \in \mathbf{R}^N$, is said to be a competitive equilibrium if

(i) $y^{0j} \in Y^j$ and $p^0 \cdot y^{0j} \geq p^0 \cdot y$ for all $y \in Y^j$, for all $j \in F$,
(ii) $x^{0i} \in X^i$, $p^0 \cdot x^{0i} \leq M^i(p^0) = p^0 \cdot r^i + \sum_{j\in F} \alpha^{ij} p^0 \cdot y^{0j}$ and $x^{0i} \succeq_i x$ for all $x \in X^i$ with $p^0 \cdot x \leq M^i(p^0)$ for all $i \in H$, and
(iii) $0 \geq \sum_{i\in H} x^{0i} - \sum_{j\in F} y^{0j} - \sum_{i\in H} r^i$ with $p^0_k = 0$ for coordinates k so that the strict inequality holds.

This definition is sufficiently general to include the equilibria developed in Theorems 7.1, 11.1, and 17.7.

12.2 First Fundamental Theorem of Welfare Economics

We are now ready to state and prove the First Fundamental Theorem of Welfare Economics. It says that every equilibrium is an optimum. A competitive equilibrium allocation is always Pareto efficient. The result is remarkable in two ways. First, it requires virtually no assumptions or mathematical structure beyond the definitions of equilibrium and efficiency and an assumption of scarcity (monotonicity). Second, it does not require convexity of tastes or technology. In addition, the proof is disarmingly simple. We start from a competitive equilibrium. That means that households are optimizing utility subject to a budget constraint and that firms are maximizing profits. We use a proof by contradiction. Suppose the theorem were false. That would mean that there is an attainable Pareto preferable allocation. Evaluate the preferable allocation at equilibrium prices. For those households whose consumptions are strictly improved at the alternative allocation, the cost of their consumption bundle must go up as well. If these more expensive bundles are attainable, then they must be more profitable as well. But that leads to a contradiction. If they are more profitable and attainable then the equilibrium allocation cannot be an equilibrium. The contradiction proves the theorem.

To prove the First Fundamental Theorem of Welfare Economics, it is useful to have the budget constraint fulfilled as an equality in equilibrium, as noted in Lemmas 7.1, 10.1, or 17.4.

Theorem 12.1 (First Fundamental Theorem of Welfare Economics) Assume C.II and C.IV. Let $p^0 \in \mathbf{R}^N_+$ be a competitive equilibrium price vector of the economy. Let $w^{0i} \in X^i$, $i \in H$, be the associated individual consumption bundles, and let y^{0j}, $j \in F$, be the associated firm supply vectors. Then w^{0i} is Pareto efficient.

Proof $w^{0i} \succeq_i x$, for all $x \in X^i$ so that $p^0 \cdot x \leq M^i(p^0)$, for all $i \in H$. This is a property of the equilibrium allocation. Consider an allocation x^i that household $i \in H$ regards as more desirable than w^{0i}. If the allocation x^i is preferable, it must also be more expensive. That is,

$$x^i \succ_i w^{0i} \text{ implies } p^0 \cdot x^i > p^0 \cdot w^{0i}.$$

Similarly, profit maximization in equilibrium implies that production plans more profitable than y^{0j} at prices p are not available in Y^j. $p^0 \cdot y > p^0 \cdot y^{0j}$ implies $y \notin Y^j$. Noting that markets clear at the equilibrium allocation, we have

$$\sum_{i \in H} w^{0i} \leq \sum_{j \in F} y^{0j} + r.$$

Note that, for each household $i \in H$, by C.IV (as emphasized in Lemmas 7.1, 10.1, and 17.4),

$$p^0 \cdot w^{0i} = M^i(p^0) = p^0 \cdot r^i + \sum_j \alpha^{ij}(p^0 \cdot y^{0j}),$$

and summing over households,

$$\begin{aligned}
\sum_{i \in H} p^0 \cdot w^{0i} = \sum_i M^i(p^0) &= \sum_i \left[p^0 \cdot r^i + \sum_j \alpha^{ij}(p^0 \cdot y^{0j}) \right] \\
&= p^0 \cdot \sum_i r^i + p^0 \cdot \sum_i \sum_j \alpha^{ij} y^{0j} \\
&= p^0 \cdot \sum_i r^i + p^0 \cdot \sum_j \sum_i \alpha^{ij} y^{0j} \\
&= p^0 \cdot r + p^0 \cdot \sum_j y^{0j} \ \left(\text{since for each j, } \sum_i \alpha^{ij} = 1 \right).
\end{aligned}$$

Suppose, contrary to the theorem, there is an attainable allocation $v^i \in X^i$, $i \in H$, so that $v^i \succeq_i w^{0i}$, for all i with $v^h \succ_h w^{0h}$ for some $h \in H$. The allocation v^i must be more expensive than w^{0i} for those households made better off and no less expensive for the others. Then we have

$$\sum_{i\in H} p^0 \cdot v^i > \sum_{i\in H} p^0 \cdot w^{0i} = \sum_{i\in H} M^i(p^0) = p^0 \cdot r + p^0 \cdot \sum_{j\in F} y^{0j}.$$

But if v^i is attainable, then there is $y'^j \in Y^j$ for each $j \in F$, so that

$$\sum_{i\in H} v^i \leq \sum_{j\in F} y'^j + r,$$

where the inequality holds coordinatewise. But then, evaluating this production plan at the equilibrium prices, p^0, we have

$$p^0 \cdot r + p^0 \cdot \sum_{j\in F} y^{0j} < p^0 \cdot \sum_{i\in H} v^i \leq p^0 \cdot \sum_{j\in F} y'^j + p^0 \cdot r.$$

So $p^0 \cdot \sum_{j\in F} y^{0j} < p^0 \cdot \sum_{j\in F} y'^j$. Therefore, for some $j \in F$, $p^0 \cdot y^{0j} < p^0 \cdot y'^j$.

But y^{0j} maximizes $p^0 \cdot y$ for all $y \in Y^j$; there cannot be $y'^j \in Y^j$ so that $p \cdot y'^j > p \cdot y^{0j}$. Hence, $y'^j \notin Y^j$. The contradiction shows that v^i is not attainable. QED

Note that the First Fundamental Theorem does not require convexity of tastes or technologies. If there is an equilibrium in a nonconvex economy (a possibility since convexity is part of the sufficient, not necessary, conditions for existence of equilibrium), then the equilibrium allocation is Pareto efficient.

Theorem 12.1, the First Fundamental Theorem of Welfare Economics, is a mathematical statement of Adam Smith's notion of the invisible hand leading to an efficient allocation. A competitive equilibrium decentralizes an efficient allocation. Prices provide the incentives so that firms and households guided by prices and self-interest can, acting independently, find an efficient allocation.

12.3 Second Fundamental Theorem of Welfare Economics

The Second Fundamental Theorem of Welfare Economics says that every Pareto efficient allocation of an economy with convex preferences and technology is an equilibrium for a suitably chosen price system, subject to an initial redistribution of endowment and ownership shares. Any desired redistribution of welfare (subject to attainability) can be achieved through

12.1.eps1

Fig. 12.1. Supporting an efficient allocation (Theorem 12.2).

a market mechanism subject to a redistribution of endowment and ownership.[1] The strategy of proof is to characterize an efficient allocation as a on the boundaries of two convex sets with disjoint interiors: the set of attainable allocations and the set of Pareto preferable allocations. The Separating Hyperplane Theorem tells us that we can run a hyperplane between them. The normal to the hyperplane is the price system that supports the efficient allocation. This is presented in Theorem 12.2. It is then a matter of bookkeeping to attribute endowments to households to allow them to support the allocation as an equilibrium. That is the corollary that embodies the Second Fundamental Theorem of Welfare Economics. This is actually a very familiar result from the Robinson Crusoe economy and is illustrated in Figure 12.1.

In proving Theorem 12.2, we will fully utilize the structure of technology and preferences, particularly convexity, developed above. The economy of Theorem 12.2 is characterized by convexity of the aggregate technology set $Y (= \sum_{j\in F} Y^j)$, convexity of preferences and consumption sets X^i, and continuity and monotonicity of preferences.

In order to prove Theorem 12.2, we will use the Separating Hyperplane Theorem. Recall:

Theorem 2.12 (Separating Hyperplane Theorem) Let $A, B \subset \mathbf{R}^N$; let A and B be non-empty, convex, and disjoint, that is, $A \cap B = \phi$. Then there is $p \in \mathbf{R}^N$, $p \neq 0$, so that $p \cdot x \geq p \cdot y$ for all $x \in A, y \in B$.

In addition, a minor lemma helps with the technical structure of the proof.

Lemma 12.1 Assume C.II, C.III, C.IV. Let $x^0 \in X^i$. Then there is $x^\nu \in X^i$, $\nu = 1, 2, 3, \ldots, x^\nu \succ_i x^0$, so that $x^\nu \to x^0$.

Proof Under C.II there is $y >> x^0, y \in X^i$ and under C.III, the sequence $x^\nu = (1-1/\nu)x^0 + (1/\nu)y$ has the property that $x^\nu \in X^i, \nu = 1, 2, 3, \ldots, x^\nu \succ_i x^0$. Trivially, $x^\nu \to x^0$. QED

Recall the definition $A^i(x^i) \equiv \{x \mid x \in X^i, x \succeq_i x^i\}$. Under the assumptions of convexity and continuity of preferences, $A^i(x^i)$ is a closed convex

[1] Note that this may require an implausible redistribution of labor endowment, that is, redistributing to one household ownership of another's labor.

set. Starting from the allocation $x^i, i \in H$, we can take the sum of sets $\sum_{i \in H} A^i(x^i)$; this sum, called A, is also a convex set and represents the set of aggregate consumptions preferred or indifferent to x^i. Consider a subset of A that includes aggregate consumptions strictly preferred to x^i (approximately the interior of A). Let us denote this set by $\mathcal{A}$, which is also a convex set. A point in $\mathcal{A}$ represents an aggregate consumption mix that can provide an allocation Pareto preferable to x^i, $i \in H$. The set of aggregate attainable allocations is the (coordinatewise) nonnegative elements of $Y + \{r\}$. We will denote this set as $B = (Y + \{r\}) \cap \mathbf{R}^N_+$, a convex set. Starting from a Pareto efficient allocation x^i, $i \in H$, under monotonicity, the sets $\mathcal{A}$ and B must be disjoint. If not, there would be an attainable Pareto preferable allocation. But this is precisely the setting where we can employ the Separating Hyperplane Theorem. The normal to the separating hyperplane is the price system that decentralizes the efficient allocation. The existence of such a price system is the import of Theorem 12.2.

Theorem 12.2 Assume P.I and C.I–C.VI. Let x^{*i}, y^{*j}, $i \in H$, $j \in F$, be an attainable Pareto efficient allocation. Then there is $p \in P$ so that

(i) x^{*i} minimizes $p \cdot x$ on $A^i(x^{*i})$, $i \in H$, and
(ii) y^{*j} maximizes $p \cdot y$ on Y^j, $j \in F$.

Proof Let $x^* = \sum_{i \in H} x^{*i}$, and let $y^* = \sum_{j \in F} y^{*j}$. Note that $x^* \leq y^* + r$ (the inequality applies coordinatewise). Let $A = \sum_{i \in H} A^i(x^{*i})$. Let $B = Y + \{r\}$. A and B are convex sets. Let $\mathcal{A} = \sum_{i \in H} \{x \mid x \in X^i, x \succ_i x^{*i}\} = \sum_{i \in H} \{X^i \backslash G^i(x^{*i})\}$, a convex set whose closure is A (by Lemma 12.1). Set $\mathcal{A}$ represents aggregate consumption bundles that can provide an allocation that is a Pareto improvement over x^{*i}, $i \in H$. $\mathcal{A}$ and B are disjoint. x^* is an element of A but x^* is not interior to A or B. By the Separating Hyperplane Theorem, there is a normal p, so that

$$p \cdot x \geq p \cdot v \text{ for all } x \in \mathcal{A} \text{ and all } v \in B.$$

By continuity of preferences and continuity of the dot product we have also $p \cdot x \geq p \cdot v$ for all $x \in A$ and all $v \in B$. By C.IV, p will be nonnegative, coordinatewise. We may without loss of generality choose $p \in P$. But $x^* \leq y^* + r(co - ordinatewise)$. So $p \cdot x^* \leq p \cdot (y^* + r)$. Then x^* minimizes $p \cdot x$ on A and $(y^* + r)$ maximizes $p \cdot v$ on B. . However, x^* is the sum of many elements, one for each of $A^i(x^{*i})$, $i \in H$, and y^* is the sum of many elements, one for each Y^j, $j \in F$. Then the additive structure of A and B implies that x^{*i} minimizes $p \cdot x$ on $A^i(x^{*i})$ and y^{*j} maximizes $p \cdot y$ on Y^j.

That is,

$$p \cdot x^* = \min_{x \in A} p \cdot x = \min_{x^i \in A^i(x^{*i})} p \cdot \sum_{i \in H} x^i = \sum_{i \in H} \left(\min_{x \in A^i(x^{*i})} p \cdot x \right),$$

and

$$p \cdot (r + y^*) = \max_{v \in B} p \cdot v = p \cdot r + \sum_{j \in F} \left(\max_{y^j \in Y^j} p \cdot y^j \right).$$

So x^{*i} minimizes $p \cdot x$ for all $x \in A_i(x^{*i})$ and y^{*j} maximizes $p \cdot y$ for all $y \in Y^j$. QED

Theorem 12.2 presents the mathematical structure we need. It says that the separation theorem can be used to find prices that support any efficient allocation. The corollary below constitutes the Second Fundamental Theorem of Welfare Economics. It says that the supporting prices introduced in Theorem 12.2 can be used, along with a suitably chosen redistribution of endowment, to support any chosen efficient allocation as an equilibrium.

For full generality, the corollary presents two possible cases of household incomes. This represents the complexity of corner solutions again. Case 1 (presumably the most common) occurs when the household expenditure at the efficient allocation exceeds the minimum level in the consumption set. Then the household is a utility maximizer subject to budget constraint. Case 2 occurs when the efficient allocation attributes expenditure to the household equal the minimum in its consumption set. In that case the household is an expenditure minimizer subject to utility constraint. Restricting attention to interior allocations would eliminate this complexity by confining attention to Case 1.

Corollary 12.1 (Second Fundamental Theorem of Welfare Economics) Assume P.I and C.I–C.VI. Let x^{*i}, y^{*j} be an attainable Pareto efficient allocation. Then there is $p \in P$ and $\hat{r}^i \in \mathbf{R}^N$, $\hat{r}^i \geq 0$, $\hat{\alpha}^{ij} \geq 0$, so that

$$\sum_{i \in H} \hat{r}^i = r,$$

$$\sum_{i \in H} \hat{\alpha}^{ij} = 1 \text{ for each j},$$

$$p \cdot y^{*j} \quad \text{maximizes} \quad p \cdot y \text{ for } y \in Y^j,$$

and

$$p \cdot x^{*i} = p \cdot \hat{r}^i + \sum_{j \in F} \hat{\alpha}^{ij} (p \cdot y^{*j}).$$

Further, for each $i \in H$, one of the following properties bolds:

CASE 1 $(p \cdot x^{*i} > \min_{x \in X^i} p \cdot x) : x^{*i} \succeq_i x$ for all $x \in X^i$ so that

$$p \cdot x \leq p \cdot \hat{r}^i + \sum_{j \in F} \hat{\alpha}^{ij}(p \cdot y^{*j}), or$$

CASE 2 $(p \cdot x^{*i} = \min_{x \in X^i} p \cdot x)$: x^{*i} minimizes $p \cdot x$ for all x so that $x \succeq_i x^{*i}$.

Proof Applying Theorem 12.2, we have $p \in P$ so that y^{*j} maximizes $p \cdot y$ for all $y \in Y^j$ and so that x^{*i} minimizes $p \cdot x$ for all $x \in A^i(x^{*i})$. We must show two properties, (1) that $\hat{r}^i$, $\hat{\alpha}^{ij}$ can be found fulfilling the above equations and inequalities, and (2) that household behavior can be characterized as utility optimization subject to budget constraint in Case 1 and as cost minimization subject to utility level in Case 2.

By attainability of the allocation, we have

$$\sum_{i \in H} x^{*i} \leq \sum_{j \in F} y^{*j} + r.$$

By Pareto efficiency of the allocation, we know that the strict inequality applies here only in redundant goods k that are desirable to no household so $p_k = 0$. Further, by weak monotonicity, C.IV, there is at least one good that is desirable and hence supports a positive price. Multiplying through by p, with the recognition of free goods, we have

$$\sum_{i \in H} p \cdot x^{*i} = \sum_{j \in F} p \cdot y^{*j} + p \cdot r.$$

But then it is merely simple arithmetic to find suitable $\hat{r}^i$, $\hat{\alpha}^{ij}$. A simple choice (one of many possible) is to let

$$\lambda_i = \frac{p \cdot x^{*i}}{\sum_{h \in H} p \cdot x^{*h}},$$

and set $\hat{r}^i = \lambda_i r$, $\hat{\alpha}^{ij} = \lambda_i$, for all $i \in H$, $j \in F$.

On the consumer side now, we wish to show that cost minimization subject to a utility constraint is equivalent to utility maximization subject to a budget constraint in Case 1. This follows from continuity of preferences. Suppose, on the contrary, there is x'^i so that $p \cdot x'^i = p \cdot x^{*i}$ and $x'^i \succ_i x^{*i}$. We will show that this leads to a contradiction. By continuity of preferences, C.V, there is an ε neighborhood about x'^i so that all points in the neighborhood are superior or indifferent, $\succeq_i$, to x^{*i}. But then some points of the neighborhood are less expensive at p than x^{*i}, and x^{*i} is no longer a cost minimizer for $A_i(x^{*i})$. This is a contradiction, and hence there can be no such x'^i. The assertion for Case 2 is merely a restatement of the property shown in Theorem 12.2. QED

The Second Fundamental Theorem of Welfare Economics represents a significant defense of the market economy's resource allocation mechanism. It says (assuming convexity of tastes and technology) that any efficient allocation of resources can be decentralized using the price mechanism, subject to an initial redistribution of endowment. This is the basis of the common prescription in public finance that any attainable distribution of welfare can be achieved using a market mechanism and lump-sum taxes (corresponding to the redistribution of endowment). On this basis, public authority intervention in the market through direct provision of services (housing, education, medical care, child care etc.) is an unnecessary escape from market allocation mechanisms with their efficiency properties. Public authority redistribution of income should be sufficient to achieve the desired reallocation of welfare while retaining the market discipline for efficient resource utilization.

12.4 Bibliographic note

The notion that competitive equilibrium and efficient allocation are closely related concepts dates back at least to Adam Smith (1776). The mathematical treatment here, emphasizing the use of separating hyperplanes rather than the differential calculus, is attributed to Arrow (1951) and is fully expounded in Koopmans (1957) and in Debreu (1959).

Exercises

12.1 Consult Exercises 2 and 3 of Chapter 7. In each of those problems, when a competitive equilibrium exists, is the resulting allocation Pareto efficient?

12.2 Consider the general equilibrium of a pure exchange economy with redistributive income taxation. Net income taxation is not on endowment, but rather on that portion of endowment (net) sold. There is a finite number of households in the set $H, i = 1, 2, 3, \ldots, \#H$. Each household i has continuous monotone convex preferences $\succeq_i$ and is endowed with resources $r^i \in \mathbf{R}^N_+$. There is an income tax at rate τ, $0 < \tau < 1$, with rebate in the amount T. Household i's after-tax income is

$$M^i(p) = p \cdot \hat{r}^i - \tau p \cdot (r^i - x^i)_+ + T,$$

where $T = (1/\#H)\tau \sum_{i \in H}[(p \cdot (r^i - x^i)_+]$. The notation $(\cdot)_+$ denotes the vector consisting of the positive coordinates of $(\cdot)$. Household i chooses consumption $x^i \in \mathbf{R}^N_+$ to optimize $\succeq_i$ subject to

$$p \cdot x^i \leq M^i(p).$$

(a) Define a competitive equilibrium in this economy.

(b) Proving existence of competitive equilibrium is a bit tricky. When a competitive equilibrium exists, is the allocation Pareto efficient? Explain.

12.3 A well-recognized problem in industrial organization and welfare economics is allocative efficiency with a natural monopoly. A natural monopoly is a firm characterized by a large nonconvexity in the production technology, hence displaying (weakly) declining marginal costs throughout the relevant range of output levels. An efficient allocation will typically include only one firm active in this market (hence it has a monopoly). Marginal cost pricing (generally characterizing an efficient market allocation) is incompatible with a market equilibrium (marginal cost is below average cost, so marginal cost pricing leads the firm to run losses). A conventional proposal to deal with this problem is as follows:

> Government should provide a subsidy to the firm (financed by nondistortionary taxation) to repay its losses. The firm should price at marginal cost. The resulting allocation is (thought to be) Pareto efficient.

(a) Why is this proposal thought to achieve a Pareto efficient allocation?

(b) Diagram a simple Robinson Crusoe two-commodity case where it will achieve an efficient allocation.

  (i) Diagram the production frontier in the case of declining marginal cost.

  (ii) Diagram an interior optimum.

  (iii) Diagram the budget line (and the lump-sum tax) supporting the efficient allocation.

(c) Show that the proposal may also support an inefficient allocation as a marginal cost pricing equilibrium.

(i) Diagram the production frontier in the case of declining marginal cost.
(ii) Diagram a corner optimum.
(iii) Diagram the budget line (and the lump-sum tax) supporting an inefficient interior allocation.

(d) Discuss. How does this relate to the Fundamental Theorems of Welfare Economics? Can local conditions (marginal equivalences, $MRS = MRT$) fully characterize efficient allocations in this problem? Why or why not?

12.4 The usual U-shaped cost curve model of undergraduate intermediate economics includes a small nonconvexity (diminishing marginal cost at low output levels). This is a violation of our usual convexity assumptions on production (P.I or PV). Consider the general equilibrium of an economy displaying U-shaped cost curves. It is possible that a general equilibrium exists despite the small violation of convexity. After all, P.I and P.V are sufficient, not necessary, conditions. If a general equilibrium does exist despite the small nonconvexity, will the allocation be Pareto efficient? Does the First Fundamental Theorem of Welfare Economics apply? Explain.

12.5 One of the assumptions used in proving the First Fundamental Theorem of Welfare Economics, Theorem 12.1, is weak monotonicity of preferences, C.IV. Show that the theorem is false without this assumption.

12.6 External effects (e.g., air pollution, water pollution, annoyance due to neighboring noise, traffic congestion) occur in economic analysis when one firm or household's actions affect the tastes or technology of another through nonmarket means. That is, in an external effect, the interaction between two firms does not take the form of supply of output or demand for input going through the market (and hence showing up in price). It would be characterized rather as the shape of one firm's available technology set depending on the output or input level of another firm. Or it might be characterized as one firm's inputs (like clean air at a tourist resort) being nonmarketed but their availability being affected by the production decisions of another firm.

Does the model of Chapter 12 treat external effects? Explain your answer. How does the treatment of externalities (or lack of treatment) show up in the specification of the model?

12.7 Describe the significance of:

(a) The First Fundamental Theorem of Welfare Economics (Theorem 12.1).
(b) The Second Fundamental Theorem of Welfare Economics (Theorem 12.2 and Corollary 12.1).

12.8 Consider an economy with two consumption goods, x and y, and one input to production L, which is inelastically supplied. Let a and k be positive constants. Production of x is by simple constant returns,

$$x = kL^x,$$

where L^x is the amount of L used as an input to x. Production of y involves a set-up cost, $S \geq 0$ (a nonconvexity),

$$\begin{aligned} &y = 0 && \text{if } L^y \leq S \\ &y \leq a(L^y - S) && \text{if } L^y > S \end{aligned}$$

where L^y is the amount of labor used as an input to y. The total labor input supplied is

$$L^x + L^y = L^0.$$

(a) Set $S = 0$. Will a Pareto efficient allocation typically be supported as a profit-maximizing competitive equilibrium (subject to a possible redistribution of household endowments)? Explain. If the answer is "no," are there special cases where an efficient allocation can nevertheless be sustained as a competitive equilibrium? Explain. A diagram may be useful.
(b) Set $S > 0$. Will a Pareto efficient allocation typically be supported as a profit-maximizing competitive equilibrium (subject to a possible redistribution of household endowments)? Explain. If the answer is "no," are there special cases where an efficient allocation can nevertheless be sustained as a competitive equilibrium? Explain. A diagram may be useful.