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Arrow, Kenneth Joseph (born 1921)

Kenneth Arrow is the author of key post-Second World War innovations in economics that have made economic theory a mathematical science. The Arrow Possibility Theorem created the field of social choice theory. Arrow extended and proved the relationship of Pareto efficiency with economic general equilibrium to include corner solutions and non-differentiable production and utility functions. With Gerard Debreu, he created the Arrow–Debreu mathematical model of economic general competitive equilibrium including sufficient conditions for the existence of market-clearing prices. Arrow securities and contingent commodities extend the model to cover uncertainty and provide a cornerstone of the modern theory of finance.

Kenneth Arrow is a legendary figure, with an enormous range of contributions to 20th-century economics, responsible for the key post-Second World War innovations in economic theory that allowed economics to become a mathematical science. His impact is suggested by the number of major ideas that bear his name: Arrow's Theorem, the Arrow–Debreu model, the Arrow–Pratt index of risk aversion, and Arrow securities.

Four of his most distinctive achievements, all published in the brief period 1951–54, are as follows:

Arrow Possibility Theorem. *Social Choice and Individual Values* (1951a) created the field of social choice theory, a fundamental construct in theoretical welfare economics and theoretical political science.

Fundamental Theorems of Welfare Economics. 'An extension of the basic theorems of classical welfare economics' (1951b) presents the First and Second Fundamental Theorems of Welfare Economics and their proofs without requiring differentiability of utility, consumption, or technology, and including corner solutions (zeroes in quantities of inputs or outputs).

The Arrow–Debreu model of general economic equilibrium. 'Existence of equilibrium for a competitive economy' (with Gerard Debreu, 1954) creates the mathematical model of a competitive economy. The article formalizes the cross-effects between markets (effect of one market's price on another's demand and supply) and provides sufficient conditions for the existence of prices allowing decentralized market-clearing general equilibrium of a market economy. This model is central to the study of markets and welfare economics; it is now a standard of the field.

Securities markets and risk-bearing. 'Le rôle des valeurs boursières pour la répartition la meilleure du risque' (1953) introduces the concept of a 'contingent commodity'. The article formalizes the role of markets, including financial markets, insurance, and the stock market, in resource allocation; it is a cornerstone of the modern theory of finance.

Personal and intellectual history

Kenneth Arrow was born in New York City on 23 August 1921. He describes his family circumstances as financially comfortable during the 1920s, but 'my father lost everything in the great depression and we were very poor for about ten years … When it came to college, my family's poverty constrained me to attend the City College' (Breit and Spencer, 1986, p. 45). Free tuition at City College of New York (CCNY) gave a generation of New Yorkers their start on success.

The searing experience of the Depression affected career ambitions. Arrow thought he should pursue the safe career of a high-school mathematics teacher. He took education courses and he had a very successful period of practice teaching in mathematics, preparing students for the New York State Regents examination. However, the roster of applicants for New York City teachers' positions was already filled.

Arrow graduated from CCNY in 1940 with the unusual combination of a mathematics major and a Bachelor of Science in Social Science. While at CCNY he studied with Alfred Tarski in a course on the calculus of relations. Arrow was a proofreader for Tarski's *Introduction to Logic* (1941). He entered Columbia University for graduate study and received an M.A. in mathematics in June 1941. Harold Hotelling, a statistician with an appointment in the economics department, was the decisive influence. Arrow notes, 'When I took [Hotelling's] course in mathematical economics, I realized I had found my niche' (Breit and Spencer, 1986, p. 45). With the inducement of a fellowship in economics, Arrow transferred to the economics department for the rest of his graduate study.

Arrow's graduate work at Columbia was interrupted by the Second World War. During the war Arrow was a weather officer in the US Army Air Corps achieving the rank of Captain, working in the Long Range Forecasting Group. Arrow's first published paper comes from that period, 'On the optimal use of winds for flight planning' (1949a). The group's principal task was to forecast the number of rainy days in air combat areas – a month in advance. The young statisticians in the Weather Division subjected the prediction techniques in use to statistical test against a simple null hypothesis based on historical data. Finding that prevailing techniques were not significantly more reliable than the null, the junior officers sent a memo to the General of the Air Corps suggesting that the group be disbanded. Six months later, the General's secretary replied on his behalf: 'The general is well aware that your forecasts are no good. However, they are required for planning purposes.' The group remained intact.

In 1946 Arrow returned to graduate study at Columbia. Harold Hotelling had by then left for the University of North Carolina's newly formed statistics department. The concern about making a living persisted. Arrow considered a non-academic career as a life insurance actuary. Tjalling Koopmans (at a Cowles Commission meeting in Ithaca, New York) advised him that actuarial statistics would prove unrewarding, saying, with characteristic reticence, 'There is no music in it.' Fortunately for economic science, Arrow followed this advice and decided to continue a research career.

In 1947 Arrow joined the (now legendary – then fledgling) research group at the Cowles Commission for Research in Economics at the University of Chicago. It seemed a golden age – all the ideas of mathematical economic theory and econometrics were being newly discovered. The close friendships and collaborations among colleagues of the Cowles Commission lasted a lifetime. Arrow describes the setting as a 'brilliant intellectual atmosphere … with eager young econometricians and mathematically inclined economists under the guidance of Tjalling Koopmans and Jacob Marschak' (Lindbeck, 1992, p. 107).

Jacob Marschak, the Cowles Commission Research Director, arranged for the Commission to administer the Sarah Frances Hutchinson Cowles Fellowship for women pursuing quantitative work in the social sciences (the Fellowship had originally specified a preference that fellows be women of the Episcopal Church of Seneca Falls, New York [reported in conversation with Jacob Marschak]). The fellows were Sonia Adelson (subsequently married to

Lawrence Klein) and Selma Schweitzer. Kenneth Arrow and Selma Schweitzer were married in 1947.

Graduate study 1946–50, through Columbia, Chicago, Cowles, RAND and Stanford, included a daunting search for a worthy dissertation topic. Prospects considered and rejected included revising and restating the Tinbergen model (Tinbergen, 1939), and revising and restating Hicks's *Value and Capital* (1939). No topic seemed worthy. Then lightning struck: Arrow invented an entire field of economics with his dissertation 'Social Choice and Individual Values'. The Columbia Ph.D., with Professor Albert Hart as dissertation advisor, was granted in 1951. As an econometrician, T. W. Anderson of Columbia (subsequently Arrow's colleague at Stanford) was called upon to pass judgement on a draft thesis unrecognizable as economics to Ken's advisors; Anderson pronounced the work sound.

The summer of 1948 and several summers thereafter were spent at the recently formed RAND Corporation in Santa Monica, California, a major centre of the newly emerging specialities of game theory and mathematical programming. In 1949 Arrow was appointed Acting Assistant Professor of Economics and Statistics at Stanford University, and rapidly became Professor of Economics, and of Statistics, with the eventual additional title of Professor of Operations Research. He moved to Harvard in 1968 (returning regularly to Stanford for summer workshops), and rejoined the Stanford faculty in 1979. He retired in 1991.

In the 1950s and 1960s at Stanford, economic theory and econometrics faculty and graduate students were located in Serra House (converted from the retirement residence of the first president of the university) under the auspices of the Institute for Mathematical Studies in the Social Sciences (IMSSS) organized under the leadership of Patrick Suppes. In his memorial remarks for his student, Walter P. Heller (1942–2001), Arrow describes the *esprit de corps*: 'Economic theory backed by serious mathematical reasoning was just beginning to be recognized...Our group of faculty and students in economic theory at Serra House ... felt ourselves a community. Not an oppressed minority, but rather a vanguard. We were taking over!'

Stanford and UC Berkeley were centres of research in statistics and economic theory. The joint Berkeley–Stanford Mathematical Economics Seminar met biweekly at alternate campuses. The Berkeley group included Gerard Debreu, Roy Radner, Peter Diamond, and Dan McFadden. Stanford's included Herbert Scarf and Hirofumi Uzawa. Uzawa came to Stanford on fellowship arranged by Arrow. Working on his own in Japan, he had written the manuscript eventually published as 'Gradient method for concave programming, II: Global stability in the strictly convex case' (Arrow, Hurwicz and Uzawa, 1958a, ch. 7). It was a successful global stability analysis of gradient adjustment, following Arrow and Hurwicz's local analysis (available to Uzawa in manuscript, published in the same volume). Arrow read the manuscript and enthusiastically invited Uzawa to accept a fellowship at Stanford.

Although the profession is now used to mathematical expression, in the 1950s and 1960s the mathematical complexity of Arrow's work was regarded as forbidding. Although Arrow was the pre-eminent economic theorist at Stanford, he was not designated to teach in the required first-year graduate microeconomic theory course; it was presumed that the treatment would be excessively abstract for this general audience. His reputation for mathematical abstraction provided the excuse for a jest when Arrow [illegible]e 1957 John Bates Clark Award of the American Economic Association (presented to a leading economist under the age of 40). At the presentation ceremony, introductory remarks were made by George Stigler, who reportedly advised Arrow, in a stage [illegible] should probably say, ''Symbols fail me''.'

Under the administration of President J. F. Kennedy, Arrow and Robert Solow served on the research staff of the Council of Economic Advisers. That was a remarkable group: Walter W. Heller, chair, Kermit Gordon and James Tobin. The Council and its staff then included three future Nobel laureates: Arrow, Solow and Tobin.

Academic travels abroad included visits to the Institute for Advanced Studies in Vienna in the summers of 1964 and 1971, and productive years at Churchill College, Cambridge, in 1963–64 and 1970, for collaboration with Frank Hahn on *General Competitive Analysis*.

To no one's surprise, Arrow received the 1972 Nobel Prize in Economic Sciences (jointly with the distinguished British economic theorist, John Hicks of Oxford). Aged 51 at the time of the award, he is (at this writing) by far the youngest recipient of the Nobel Prize in Economics.

Testimony to Arrow's qualities as a dissertation advisor, a teacher of the next generation of economists, is abundant. The flurry of former students volunteering to contribute to the Festschrift by Heller, Starr and Starrett (1986) was overwhelming. The most personal tribute is the number of leading colleagues whose children have studied with Arrow. Jacob Marschak's son Thomas Marschak and Walter W. Heller's son Walter P. Heller wrote their doctoral dissertations with Arrow as principal advisor. Any list of Arrow's students (dissertation advisees, postdocs, and so forth) is a partial listing. They are numerous and are enthusiastically devoted to him, playing leading roles in academic and research economics. A selection includes: Theodore Bergstrom (UC Santa Barbara), David Bradford (Princeton University), Michael Bruno (Hebrew University, Bank of Israel), Graciela Chichilnisky (Columbia University), Peter Coughlin (University of Maryland), John Geanakoplos (Yale University), Louis Gevers (Université de Namur, Belgium), John Harsanyi (UC Berkeley), Walter P. Heller (UC San Diego), Peter Huang (University of Minnesota Law School), Takatoshi Ito (University of Tokyo), Jean-Jacques Laffont (Université des Sciences Sociales, Toulouse, France), Robert Lind (Cornell University), Thomas Marschak (UC Berkeley), Eric Maskin (Institute for Advanced Study, Princeton), Roger Myerson (University of Chicago), Hajime Oniki (Osaka-Gakuin University, Osaka, Japan), Heraklis Polemarchakis (Brown University), Karl Shell (Cornell University), Ross Starr (UC San Diego), David Starrett (Stanford University), Nancy Stokey (University of Chicago), Laurence Weiss (Goldman Sachs Corp.), Ho-Mou Wu (National Taiwan University), and Menahem Yaari (Hebrew University, Jerusalem).

A range of stories depict Arrow as a legendary larger-than-life figure:

'Arrow is personally accessible and unpretentious, addressed as ''Ken'' by students, colleagues, and staff… Arrow thinks faster than he – or anyone else – can talk. Conversation takes place at such a rapid pace that no sentence is ever actually completed' (Heller, Starr and Starrett, 1986, v. 1, p. xvii). The breadth of Arrow's knowledge is repeatedly a surprise, encompassing Chinese art, English history and the works of Shakespeare. At the 80th birthday celebration, Eric Maskin related the following example:

> On almost any subject arising in conversation, Arrow turns out to know a lot more than you do. Tired of being repeatedly shown up by their senior colleague, a group of junior faculty once concocted a plan. They first read up thoroughly on the most arcane topic they could think of – the breeding habits of gray whales. On the appointed day they gathered in the coffee room and waited for Ken to come in. Then

> they started talking about whales, concentrating on the elaborate theory of a marine biologist named Turner on how gray whales found their way back to the same breeding spot year after year. Ken was silent … they had him at last! With a sense of delicious triumph, they continued to discuss whales, and Ken looked more and more perplexed. Finally, he couldn't hold back: 'But I thought that Turner's theory was entirely discredited by Spencer, who showed that the hypothesized homing mechanism couldn't possibly work.'

Arrow's presence in seminars is distinctive. He may open his (copious) mail, juggle a pencil, seem inattentive. He will then make a comment demonstrating that he is several steps ahead of the speaker. He will make clear that the history of economic thought includes abundant antecedents (which he can readily cite from memory) for the issues under discussion.

Social Choice and Individual Values: The General Possibility Theorem

Social Choice and Individual Values was Arrow's doctoral dissertation, published as a Cowles Commission monograph. There are very few new ideas in economics. Arrow's General Possibility Theorem is as novel and fundamental as they come. The paradox of voting (cyclic majorities) appears to have been well-known, though not well formalized; Arrow (1951a) and Duncan Black (1948) both take it as understood. A review of the literature shows that it is attributable to Condorcet (1785). The paradox – intransitivity of choice from majority vote based on voters with transitive preferences – can be stated simply.

Think of three voters trying to decide by majority vote among three possibilities, A, B, and C. Each of the individual voters has transitive (rational) preferences. Voter 1 prefers A to B and prefers B to C. Voter 2 prefers B to C and C to A. Voter 3 prefers C to A and A to B. Then there is a majority of voters preferring A to B (voters 1 and 3), and a majority preferring B to C (voters 1 and 2). If group decision-making is also transitive (rational), then the group should prefer A to C. But just the opposite occurs; there is a majority preferring C to A (voters 2 and 3). Despite the transitivity of individual preferences, the group preference on pairs of alternatives, as expressed by majority vote, is intransitive (irrational).

Arrow's General Possibility Theorem (also known as 'Arrow's Theorem', the 'Arrow Possibility Theorem', or the 'Arrow Impossibility Theorem') shows that the paradox is not merely an anomaly but intrinsic to group decision-making. The theorem has been a focus of vigorous study for generations. An elegant proof in Sen (1986) is particularly striking since it is framed as a generalization of the Condorcet paradox.

The Possibility Theorem suggests four reasonable criteria for a group decision-making mechanism, all of which are fulfilled by majority voting (assume at least three possible choices and at least three voters):

1. *Unrestricted Domain*. The decision-making mechanism can accommodate all logically possible preferences on the available choices.
2. *Pareto Principle*. If everyone prefers one alternative over another, the group decision should have that preference as well.
3. *Independence of Irrelevant Alternatives*. In choosing between any two alternatives, group decision-making takes account only of individual pref-

erences on those alternatives; preferences on a third possibility do not enter the choice between those two.
4. *Non-dictatorship*. There is no single person whose preferences will always be followed by the group decision-making mechanism.

The Possibility Theorem says that no decision-making mechanism that fulfils all four of the above conditions results in transitive (rational) group choices based on transitive (rational) individual preferences. The Condorcet paradox is not merely an anomaly. It is unavoidable. It represents a fundamental defect in group decision-making.

Each of the four above conditions is essential to the theorem; there are examples of transitive group decision-making mechanisms that fulfil any three but not four. Of the four, the most controversial is Independence of Irrelevant Alternatives; it prevents voluntary misstatement by a voter of his preferences from being an attractive strategy (overstating dislike of a third option to make a preferred one of two succeed in a weighted voting scheme).

At the time *Social Choice and Individual Values* was published, the logic of group decision-making was not even recognized as an economic issue. Since then there has been an overwhelming blossoming of the 'social choice' field. It is a topic for the *Handbook of Mathematical Economics*; thousands of journal articles deal with it; every graduate student in economics is introduced to it. Kenneth Arrow created the field by formalizing a result that says the object of the field is unachievable.

The book also had a significant impact in a second direction: treating economic theory as an axiomatic logical field rather than as a sphere of calculation. *Social Choice* was one of the first essays, certainly the first monograph, to treat economics with the same generality and logical rigour as classical geometry. This approach was to be repeated in the next of Arrow's several major works in general equilibrium theory and classical welfare economics.

How did Arrow come to develop this structure? It was during the first summer, in 1948, at RAND that several strands of thought came together. The Condorcet paradox of cyclic majorities was common knowledge (though not the attribution to Condorcet). Independently of Duncan Black (1948), Arrow developed the restriction of individual preferences to the single-peaked format as a solution, but then realized that he'd been scooped when he read Black's result in the *Journal of Political Economy*. He was aware of the ambiguity in describing the optimizing policy of a business firm under uncertainty: profit maximization is no longer well-defined and majority voting of shares is subject to the Condorcet paradox. Arrow's techniques of logical formalization were ready. As a high-school student he had read Russell's *Introduction to Mathematical Philosophy*; at CCNY he became familiar with Tarski's *Introduction to Logic* and the calculus of relations. With that preparation, it was obvious that the indifference curve approach used by economists was a form of a logical ordering. Axiomatic treatment came naturally.

RAND was the centre of the developing field of game theory, which was being used to formalize discussions of strategic behaviour in international relations. During a coffee break the logician Olaf Helmer posed the following problem. Game theory supposes rational strategic behaviour among optimizing agents. The maximand of an individual may be well-defined, perhaps as a utility function; but what is the maximand of a country? Arrow replied that a Bergson social welfare function should represent a country's maximand. That set him to work. Demonstrating that his answer to Helmer was fundamentally and necessarily inadequate is the meaning of the Possibility

Theorem. Arrow started the inquiry by looking at a variety of group decision-making mechanisms. They all looked wrong; either they led to intransitivity or they violated the Independence of Irrelevant Alternatives, so that preferences for an alternative that was out of the running nevertheless entered the group's decision. He was led to formalize the conditions of group decision-making, reflecting a long-standing interest in axiomatic reasoning. 'The development of the theorems and their proofs then required only about three weeks, although writing them as a monograph took many months' (1983a, p. 4).

Extension of the fundamental theorems of welfare economics

In the 1940s welfare economics in mathematical form (the relationship of market equilibrium to economically efficient allocation) was very much a matter of the calculus (Samuelson, 1947). Marginal rates of substitution (ratios of marginal utilities) were equated to marginal rates of transformation (ratios of marginal products of factors) which were equated to price ratios. This is a sound viewpoint so long as the underlying functions are differentiable and the quantities of goods and factors are in a range where they can be varied. Arrow's view was that there is a fundamental weakness to this approach in the presence of non-negativity constraints on quantities. It works only when quantities are strictly positive. That is, the calculus doesn't treat corner solutions. But almost every practical economic solution is a corner solution: it is rare to find that all quantities of all possible goods and all possible inputs are used in strictly positive quantities. This is particularly true when differing qualities or varieties of similar goods are treated distinctly (white, sourdough, and rye breads are distinct commodities, as are luxury and efficiency apartments). There must be a welfare economics that includes corner solutions; it must be possible to present welfare economics without the calculus.

Arrow attributes his insight to a seminar presentation on the fundamental theorems of welfare economics given by Paul Samuelson at the University of Chicago, in Samuelson's style using the calculus (1983b, p. 14). The diagrams that illustrated the equations depicted a separating hyperplane. Arrow had learned of the fundamental role of convexity and the separating hyperplane theorem at RAND in the summer of 1948. The result of these reflections is 'An extension of the basic theorems of classical welfare economics' appearing in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*. The conference was held in the summer of 1950 in Berkeley, and the proceedings appeared a year later. There, the First and Second Fundamental Theorems of Welfare Economics are stated in terms of real analysis and convex sets, without the use of the calculus and including corner solutions.

At the level of the firm and the household, characterizing optimizing behaviour at corner solutions is the job of the Kuhn–Tucker Theorem. In a case of simultaneous discovery of related ideas, that theorem was first publicly presented at the same Berkeley Symposium (Kuhn and Tucker, 1951).

First Fundamental Theorem of Welfare Economics: Every competitive equilibrium allocation is Pareto efficient. This result does not require convexity of tastes or technology, though convexity may be useful in establishing the existence of equilibrium prices.

Second Fundamental Theorem of Welfare Economics: In an economy with convex technology and preferences, every Pareto-efficient allocation can be sustained as a competitive equilibrium with appropriate prices subject to a *redistribution of ownership shares in firms and redistribution of endowment (except that some low-income households may be expenditure minimizers subject to utility constraint, rather than utility maximizers subject to budget).*

Neither of these results depends on positivity of quantities or on differentiability of the functions or relations. The generality of the results, the use of a formal mathematical structure of assumptions, theorems and proofs was again novel. It meant that economics was becoming closer to formal mathematics.

General equilibrium theory

In the early 1950s, Arrow (at Stanford) pursued, largely by correspondence, joint work on general equilibrium theory with Gerard Debreu, who was then at the Cowles Commission in Chicago. The theory of general economic equilibrium recognizes that the economy is an interactive system. Decisions and prices in one market have a direct impact on supply and demand in other markets. The question Arrow and Debreu treated is: under what (sufficiently general and formalized) conditions can there be prices so that all markets simultaneously clear? This issue is known as 'the existence of economic general equilibrium'. The term 'general' equilibrium refers to the many markets simultaneously clearing, as opposed to 'partial' equilibrium where a single market is considered in isolation. Moreover, the theory allows – or forces – the theorist to formulate relatively complete models of the economy. The result of these inquiries has been an intellectual revolution and an intellectual foundation for market economics. A half-century after it was introduced to economics, the Arrow–Debreu model is the cornerstone and workhorse of our theory of markets and resource allocation.

Abraham Wald, with whom Arrow had studied at Columbia, had written several papers in the field (while in Vienna in the 1930s before emigrating to avoid the Nazi takeover) but had run up against fundamental mathematical difficulties (Wald, 1934–35, 1936). He explained to Arrow that the problem was 'very difficult', advice that was enough to discourage the young economic theorist for some years. It was the recognition by Arrow and Debreu of the importance of using a fixed point theorem that led to major progress in this area. (Credit for independent discovery of the importance of fixed point theorems in this context is due to Lionel McKenzie, 1954. The use of a fixed point theorem for demonstrating the existence of an equilibrium [of a game] was pioneered by John Nash, 1950. See Debreu, 1983).

Arrow describes his early thoughts on the subject and the interaction with ideas current at the time (particularly the Nash equilibrium of N-person games) thus:

> My original approach, for what it is worth, was to formulate competitive equilibrium as the equilibrium of a suitably chosen game. The players of this fictitious game were the consumers, a set of 'anticonsumers' (one for each consumer), producers, and a price chooser. Each consumer chose a consumption vector, each anticonsumer a non-number (interpretable as the marginal utility of income), each firm a production vector, and the price chooser a price vector on the unit simplex. The payoff to a consumer was the utility of his consumption vector plus the budgetary surplus (possibly negative, of course) multiplied by the anticonsumer's chosen number. The payoff to an anticonsumer was the negative of the payoff to the corresponding consumer. The payoff to the firm was profit and to the price chooser

> the value of excess demand at the chosen prices. This is a wel[illegible]e. The existence of equilibrium does not follow mechanically from Nash's theorem, since some of the strat[illegible]ains are unbounded.
> Debreu and I sent our manuscripts to each other and so discov[illegible] common purpose. We also detected the same flaw in each other's work; we had ignored the possibility of discontinuity when prices vary in such a way that some consumers' incomes approach zero. [The possibility of discontinuity in demand at incomes where household consumption is on the boundary of the possible consumption set is known as the 'Arrow corner'.]. We then collaborated, mostly by correspondence, until we had come to some resolution of this problem. In the main body of the work we followed more closely Debreu's more elegant formula- based on the concept of generalized games, which eliminated the need for 'anticonsumers.' (1983b, pp. 58–9)

The papers of Arrow and Debreu (1954) and McKenzie (1954) were presented to the 1952 meeting of the Econometric Society. Publication of 'Existence of equilibrium for a competitive economy' repre[illegible]ndamental step in the revision of economic analysis and modelling, demonstrating the power of a formal axiomatic approach with rela[illegible]nced mathematical techniques. The approach of the field is revolutionary: it fundamentally changes our way of thinking. Once we see things this way, it is hard to conceive of them otherwise.

Sufficient conditions for the existence of market-clearing prices – consistent with one-another – for N distinct commodities are: (a) demand and supply are continuous as a function of prices, and (b) Walras's Law. These properties are derived from fundamental assumptions on the structure of preferences and endowments of households and the technology of firms. The theory is general enough to include point-valued and (convex) set-valued demand and supply.

Debreu's *Theory of Value* (1959) made the Arrow–Debreu general equilibrium model accessible to the wider profession. The implications for economic theory as a discipline were multifaceted: *general* equilibrium, treating all markets as interacting together, became systematic; the axiomatic method was set firmly in place as part of economic theory. Economic theory could be as precise and logically demanding as geometry. The potential of formal theory to generalize could be brought to bear. The Arrow–Debreu treatment proved, with full mathematical rigour, that any economy fulfilling the model's clearly and generally specified assumptions would produce its specified results.

A number of articles (principally co-authored with Leonid Hurwicz, 1958b, 1959) treat the stability of general equilibrium. Though Arrow and Debreu (1954) establishes the existence of market clearing prices, it does not derive 'equilibrium' as the rest point of a dynamic system. The stability question focuses on how a price adjustment system will lead to market clearing prices. Since prices in each market (at least potentially) enter into the excess demands of all markets, there is plenty of room for price adjustments to go awry. This body of literature sorts out and proves sufficient conditions for adjustment to be successful. Bottom line: a sufficient condition is that other markets do not excessively interfere with excess demands on any single market; if the principal determinant of excess demands for each good is the price of that good, then price adjustment to market clearing will be successful.

The effect of the introduction of the Arrow–Debreu model on economic theory has been overwhelming. Every graduate-level textbook in microeconomic theory discusses it. Whole classes of economic theorists describe their speciality as 'general equilibrium theory'. In the 15 years following publication of Theory of Value, a major focus of pure theory was understanding and extending the model. This included its relationship to bargaining (Debreu and Scarf, 1963), to large economies (Aumann, 1966) and to computing general equilibrium prices (Scarf and Hansen, 1973). It was further elaborated by Arrow and Hahn (1971a).

Contingent commodities

Part of the power of mathematics is generalization. If you've solved a problem once, you don't have to solve it again – even in different circumstances if you can show that the previous treatment applies. This was the brilliantly simple insight in the creation of the concept of 'contingent commodity'.

Arrow's thought had been influenced by Hicks's *Value and Capital*, including understanding the power of defining a commodity to include specification of time and location, and by L. J. Savage's lectures on mathematical statistics at Chicago, including a notion of the 'state of the world' as defining a random variable. (The 'state of the world' concept for defining a random variable is attributable to Kolmogorov [1933]). It was a fundamental step to combine these notions so that a commodity might be defined by what it is, where and when deliverable, *and by the 'state of the world' in which it is deliverable*.

By redefining a 'commodity' in this way as a 'contingent commodity', the complete structure of the Arrow–Debreu model of general equilibrium and economic efficiency could be applied. This is now typically described in the literature as 'a full set of Arrow–Debreu futures contracts'. The concept of an efficient (or 'optimal') allocation of risk-bearing is immediately evident as a consequence of the modelling structure. The next step is to suggest a security contract contingent on the state of the world payable in money – to economize on the number of actively traded commodities – now known as an 'Arrow security' or 'Arrow insurance contract'. This has been an extremely powerful concept, allowing researchers to formulate their ideas clearly; the Arrow security is a staple of 21st century theoretical finance.

The paper 'Le rôle des valeurs boursières pour la répartition la meilleure des risques', originally written in English, was translated into French for a conference at Centre National de Recherche Scientifique, Paris, in June 1952. Other conference participants included Jacob Marschak, Maurice Allais, L. J. Savage, Milton Friedman and Pierre Massé. It was published in French in *Econométrie* and the original English version appeared (as a 'translation') a decade later in *Review of Economic Studies*, after the notions had been introduced to English-speaking readers in *Theory of Value*.

Individual behaviour towards risk, economics of medical care, learning by doing

Treatment of uninsurable risk (where contingent commodities and Arrow securities are not available or correctly priced) has been a focus of Arrow's work for decades. It appears in the *Collected Papers*, the *Aspects of the Theory of Risk Bearing (Yrjo Jahnsson lectures)* (1965a), and in *Essays in the Theory of Risk Bearing* (1971b). These essays provide for many readers the most systematic treatment available of the statement and proof of the Expected Utility Theorem, derivation of the Arrow–Pratt risk aversion index, and a systematic framework for considering decision-making in an uncertain world.

Several papers (1963, 1965b) treat the economics of medical care, a setting where uncertainty, information as a scarce resource, and insurance all play a part. An element of the contribution is to state the issues in an abstract analytic economic framework. This reminds economists of why these problems are not textbook economics, and reminds non-economists that the economics textbook is useful. The historical setting in which these articles were written is pre-1990, that is, before health maintenance organizations (HMOs) became popular, when the principal form of medical insurance available was fee for service. They contain several insights (probably not unique to or first from Arrow, but effectively presented). For example, medical needs are uncertain so medical insurance is not merely a form of payment but is a response to risk. Again, medical insurance reduces the marginal cost of care as seen by the patient below actual cost, encouraging increased use (moral hazard consequence of insurance). Finally, medical care is distinct (but not unique) among commodities in that the decisions to incur care and the form that it should take are made to a large extent by the provider (the medical doctor) who is paid for providing care rather than by the buyer (patient). There is a resulting conflict of interest and reliance on professional norms. Arrow's treatment of the doctor-patient relationship as a seller-buyer interaction is an early appearance in the literature of the conflict we now recognize as the 'principal-agent problem' with an attendant family of issues.

In the 18th century Adam Smith noted that one of the benefits of specialization in production was that workers at specialized tasks learned how most effectively to perform them. Arrow's 'Economic implications of learning by doing' (1962) reflects in part the temper of the time – economic growth and growth models were a principal focus of theory and policy. In addition, it is a leap several decades ahead in growth theory. In contrast to growth models in the 1960s, it presents endogenous growth, a research topic that became an active focus decades later (Romer, 1994). The study brings together two apparently disparate strands of economic modelling: technical change and the theory of external effects. The benefits of production in a particular line of work include not only output but the greater experience of the firm and the workforce in production. Through production, workers and firms learn how to produce more with fewer inputs. To the extent that this knowledge is inappropriable or non-marketable, it provides an external benefit to the economy. This on-the-job experience will typically be underprovided relative to an economically efficient allocation.

Optimal programming, control theory, mathematical statistics, racial discrimination, and the CES production function

In 16 books (not including the *Collected Papers*) and 250 technical articles, there are significant contributions to a breadth of issues in economics, mathematical programming and public policy. There's even some mathematical statistics (with Blackwell and Girshick, 1949b).

One of the most useful – to other economists – is 'Capital-labor substitution and economic efficiency' by Arrow, Chenery, Minhas and Solow (1961). It introduced the constant-elasticity-of-substitution (CES) production function, spawning an immense empirical literature.

Public Investment, the Rate of Return, and Optimal Fiscal Policy and several papers with Mordecai Kurz (1970) introduced control theory to the theory of the firm, to the theory of the household, and to public finance. A variety of books and articles treat mathematical programming and optimal inventory policy.

Several papers formally model racial discrimination in employment (1973). This is a tricky problem, and not merely because it is politically controversial. Pure microeconomic theory would suggest that there should be no racial discrimination by rational profit-maximizing employers; significant discrimination should result in below-market wage rates for the discriminated-against workers with resultant extra incentive for employers to hire them. How then can an economic model of optimizing behaviour explain the prevalence of racial discrimination? The answers (based on the racial views of employers, employees, customers) provide clues to locating the points of leverage that may lead to amelioration or policy.

What have we learned?

Arrow, along with Debreu, was a decisive figure in introducing the axiomatic method to economic theory. *Social Choice and Individual Values* and 'Existence of equilibrium for a competitive economy' fundamentally changed the agenda of economic theory. Formal logical reasoning and formal statement of assumptions and conclusions became the standard of pure theory (Suppes, 2005). The axiomatic method need not be a straitjacket. Arrow's less formal work demonstrates the role of insight: observing actual economic activity and asking 'why?', where the acceptable class of answers reflects underlying principles of economic analysis. The result is a rich understanding of the nuance and power of economics.

Celebrations

Dedicated colleagues and students have done their best to show adulation and gratitude to Arrow. There has been a succession of public celebrations.

On Arrow's 65th birthday in August 1986, an immense birthday conference and party, known as the 'Arrowfest', took place at Stanford. It reunited colleagues and students from all over the world. There were two days of conference papers and testimonial remarks. A three-volume Festschrift was presented (on time) (Heller, Starr and Starrett, 1986), including papers by 35 of Arrow's students and colleagues. Among the contributing authors were three (eventual) Nobel laureates: John Harsanyi, Amartya Sen and Robert Solow. The observance included a gala dinner with testimonial remarks and an expression of thanks from Arrow.

To observe the 70th birthday, the celebration was at the doctoral alma mater, a conference and social gathering in October 1991 titled 'Columbia Celebrates Arrow's Contributions'. The Festschrift volume (Chichilnisky, 1999) included papers by 22 colleagues and students. The 70th birthday was also the occasion of formal retirement from active faculty status at Stanford. That rite of passage was observed with a reception, including testimonials from colleagues, among them the senior colleagues who had been clever enough to recruit Arrow to Stanford two generations earlier. Stanford's Arrow Lecture Series was initiated, annually inviting distinguished speakers in economic theory in Arrow's honour.

A 40th anniversary party for general equilibrium theory was held in June 1993 at Center for Operations Research and Econometrics (CORE) of the Université Catholique de Louvain in Louvain-la-Neuve, Belgium. For several days and nights hundreds of professors, researchers, and students from around the world presented papers, discussions, and reminiscences of the speciality they had pursued for years. At the centre of the celebration were the 20th century founders of the field, Kenneth Arrow, Gerard Debreu and Lionel McKenzie.

There was a happy coincidence in 2001, when the 50th anniversary of *Social Choice and Individual Values* approximately coincided with Arrow's 80th birthday. A panel discussed the book's impact over the previous half century: Pat Suppes (Stanford University) on philosophy, John Ferejohn (Stanford University) on political science, and Eric Maskin (Institute for Advanced Study) on economics. The gathering included Professor Ted Anderson, who was at Columbia when *Social Choice* was submitted as Arrow's dissertation.

A dinner that evening featured moving toasts of appreciation by colleagues from around the world and presentations by Arrow's sons, Andy and David. The conclusion – sending the audience out singing into the evening – was the ad hoc musical group, the Economy Singers, singing advice to rising young economists: 'Brush Up Your Arrow, Start Quoting Him Now.'

To many students and colleagues, Kenneth Arrow is a source of inspiration and a focus of friendship and respect:

> … an inspirational teacher and colleague ... The intellectual standards he set and the enthusiasm with which he approaches our subject are surely part of all of us ... Those of us who have had a chance to know him well are particularly fortunate. We are far richer for the experience. (Heller, Starr and Starrett, 1986, vol, 1, pp. xi, xvii)

Ross M. Starr

See also

<xref=xyyyyyy> Arrow's Theorem;
<xref=C000533> computable general equilibrium (CGE) models;
<xref=xyyyyyy> Cowles Commission;
<xref=D000238> Debreu, Gerard;
<xref=xyyyyyy> general equilibrium (recent developments);
<xref=xyyyyyy> general equilibrium (testable implications);
<xref=G000172> general equilibrium with incomplete markets;
<xref=xyyyyyy> Sen, Amartya;
<xref=U000005> uncertainty;
<xref=xyyyyyy> voting games.

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Index terms

Index terms not found: