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Market-makers' supply and pricing of financial market liquidity

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Abstract

The bid/ask spread (inverse of liquidity) in turbulent financial markets—modeled theoretically—adjusts to market-makers' average costs. Market liquidity declines (spread increases) with increasing absolute value of market-makers' security inventories and volatility of security price and order flow.

Keywords: Liquidity; Bid/ask spread; Order flow; Market-maker; Inventory

JEL classification: G12

1. Introduction: market-makers and market liquidity

Liquidity in a financial market—the ability to absorb smoothly the flow of buying and selling orders—comes from the depth of buyers and sellers in the market and from market-makers (specialists on the organized exchanges). The market-makers' business is acting as intermediaries: buying from public sellers and selling to public buyers. The market-maker provides market liquidity and makes his profit from the difference between his buying and selling (bid and ask) prices. One simple measure of the liquidity of a financial market at any moment is the bid/ask spread. Liquid markets will be characterized by a narrow spread, illiquid markets by a wide spread.

In actual markets, the behavior of the bid/ask spread varies with market conditions. Increasingly turbulent markets, such as those around October 19, 1987, late August through October 1998, and April 10–14, 2000, are accompanied by significant expansion of the bid/ask spread. In this paper we develop a simple model of market-maker pricing that gives an elementary explanation of the variation

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in market liquidity with variation in market conditions. The bid/ask spread is presented as a proportion of the security's price. The bid/ask spread is the price of the market-maker's services; it adjusts in equilibrium as the result of the market-maker's optimizing behavior. The market-maker is characterized as a monopolistic competitor, a profit maximizer subject to an (average) zero-profit condition due to the threat of entry. The market-maker's cost is the opportunity (or borrowing) cost of his capital; his marginal cost is (weakly) increasing as his financial position is extended (an increasing long or short position), which is either financed by debt or by placing increasing equity at risk.

The principal result of this model is that the bid/ask spread is an increasing function of the size of the market-maker's net long or short position. The bigger the inventory of the market-maker, the higher will be his average cost of capital, and the wider will be the bid/ask spread. Hence, imbalance in the buy–sell mix of order flow is priced in the bid/ask spread. In the case of a capital market averse to the risk of possible market-maker insolvency, the market-maker faces increasing marginal cost in the form of an interest rate increasing in the size of his inventory position. Hence, increasing asset price risk and order flow volatility also increase the market-maker's average cost (by Jensen's inequality) and the equilibrium bid/ask spread.

A distinctive element of the treatment presented here is the simplicity of the market-maker's problem. He acts primarily as a reseller of a good subject to the risk of price variability. The model ignores issues of differential information (informed versus uninformed traders) taking them to be independent of the volume and volatility considerations (particularly for the market as a whole) that the present study emphasizes. The model also separates the problem of finding equilibrium market price from the problem of setting (percentage) bid/ask spread and focuses solely on the latter. The model provides a simple explanation for declining market liquidity during periods of price volatility and of order flow imbalance (either for the market as a whole or for a single security).[1] In this setting, the market-maker typically accumulates a large net position in the security he specializes in; the market-maker buys (sells) when the public sells (buys). The significantly increased security inventory position leads to increased average cost which is then priced in the bid/ask spread.

2. Modeling the market-maker

We consider the market-maker for a single security whose price at date t is denoted P_t, conceived as the midpoint of bid and ask prices. The evolution of price over time is exogenous. The market-maker sets a symmetric proportional spread S_t (equal to half the bid/ask spread) at date t representing his price markup for the ask price and the markdown for the bid price. At date t, he faces a (long, buying) demand volume V_t^{l} and a (short, selling) supply volume V_t^{s}. The market-maker's position (net holding) of the security may be positive or negative. At the start of date t, we denote the position as N_t. The market-maker's position in the security evolves over time, increased by his purchases and reduced by his sales at t, i.e. $N_{t+1} = N_t - V_t^{\mathrm{l}} + V_t^{\mathrm{s}}$.

The market-maker passively accepts all orders to buy and sell. His only strategic action is to adjust

[1] For example, the *Wall Street Journal* of April 17, 2000, reported ''by the end of the day [April 14, 2000], it seemed as if the only buyers were NASDAQ market makers and Big Board floor specialists, who have to buy with their own capital when no other buyers can be found. After the NASDAQ's unprecedented Monday-through-Friday losing streak, market makers are holding three times as much stock as usual . . . the specialist community is stretched.''

the spread S_t. Inventory (of the security he trades) enters his choices of S_t because the costs of carrying the inventory need to be covered by the revenues from making the market.

The market-maker starts period t with a cash position M_t, carried over from the previous period. In conducting business at date t, the market-maker incurs costs C_t. We denote the market-maker's net asset value position (*not* his profit) at the start of date t as Π_t. The value of the market-maker's position at the beginning of t then is $\Pi_t = P_t N_t + M_t$. At $t+1$, the value of the position is $\Pi_{t+1} = P_{t+1} N_{t+1} + M_{t+1}$. The market-maker's cash position evolves as

$$M_{t+1} = (1 + S_t)P_t V_t^{\mathrm{l}} - (1 - S_t)P_t V_t^{\mathrm{s}} + M_t - C_{t+1}.$$

There are two components in C_{t+1}, the cost of providing market-making services. One is the direct cost of trading: order taking, execution, and record keeping. The other component is the cost of carrying an inventory of the security in which the market-maker trades. For simplicity the first component is ignored here; since we are interested in variation in the bid/ask spread and variation in the market-makers' average costs, we ignore this portion of marginal cost (assumed to be constant).[2]

3. Market-makers' pricing: a zero-profit condition

Without modeling explicitly the competitive structure of market-making, we assume a zero-profit condition. In the case of NASDAQ, this represents the outcome of ease of entry into market-making. On the NYSE, this may be taken to represent the notion of the cost of maintaining an orderly market—or a normative ceiling (not necessarily zero) on specialist profits.

The zero-profit condition implies[3]

$$E_t[\Pi_{t+1}] = E_t[M_{t+1} + P_{t+1}N_{t+1}] = \Pi_t = M_t + P_t N_t.$$

On average, the market-maker assumes no net position on the securities in which he is making a market; expected sales and purchases are on average equal. Assume the distribution of the volume of buy and sell order to be the same, with mean V_0 and variance σ_v^2. Then

$$E_t[M_{t+1}] = (1 + S_t)P_t V_0 - (1 - S_t)P_t V_0 + M_t - E_t[C_{t+1}] = 2S_t P_t V_0 + M_t - E_t[C_{t+1}].$$

Now assume that the distribution of price and volume at $t+1$ are not correlated. Further, we assume a martingale condition, that the expected mean of the security price at $t+1$ is equal to the price at t. Then

$$E_t[P_{t+1}N_{t+1}] = E_t[P_{t+1}]E_t[N_{t+1}] = P_t(N_t + V_0 - V_0) = P_t N_t.$$

Therefore, the zero-profit condition implies

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[2]Conceivably, the cost function can differ depending on whether the market-maker carries a long or short position, though we do not treat this possibility below.

[3]This expression could be presented including a time discount factor, without changing the character of the results.

$$E_t[M_{t+1} + P_{t+1}N_{t+1}] = 2S_tP_tV_0 + M_t - E_t[C_{t+1}] + P_tN_t = M_t + P_tN_t,$$

or

$$E_t[C_{t+1}] = 2S_tP_tV_0.$$

At the market equilibrium, fulfilling a zero-profit condition, the markup spread (half the bid/ask spread) is

$$S_t^* = E_t[C_{t+1}]/2P_tV_0.$$

This expression is the cornerstone of this line of research. The market-maker adjusts the bid/ask spread at any moment to cover expected (variable average) costs at expected trading volume. The market-maker pursues expected average cost pricing in a variable stochastic environment.

4. The quadratic cost case and the linear absolute value cost case

In order to derive a prediction from the pricing model above, we must specify the form of the cost function C_{t+1}. How does the size of the market-maker's net position, in the security in which he makes a market, affect his average costs? Holding the security inventory N_{t+1} requires financing, and the average cost of capital may vary with the size of the market-maker's inventory position.[4] When a market-maker relies on self-financing, the cost of carrying inventory is the opportunity cost of capital; a quadratic cost function can be interpreted as reflecting the market-maker's risk aversion. Alternatively, if the market-maker's capital is debt-financed, an increasing risk premium, represented as a quadratic cost function, may be added to interest rates on lending to a market-maker whose position is increasingly leveraged. Thus we suggest the specifications:

Case 1. (quadratic cost) $C_{t+1} = a(P_{t+1}N_{t+1})^2$. Then

$$E_t[C_{t+1}] = aE_t[((P_t + \Delta P_{t+1})N_{t+1})^2] = a(P_t^2 + \sigma_p^2)E_t[(N_t + V_{t+1}^1 - V_{t+1}^s)^2]$$
$$= a(P_t^2 + \sigma_p^2)(N_t^2 + 2\sigma_v^2 + 2V_0^2).$$

Thus

$$S_t^* = \frac{a}{2P_tV_0}(P_t^2 + \sigma_p^2)(N_t^2 + 2\sigma_v^2 + 2V_0^2).$$

Hence, in the quadratic cost case, the market-maker's bid/ask spread varies positively with price risk, σ_p^2, with trading volume risk, σ_v^2, and the size of the market-maker's trading inventory exposure N_t. As markets become more volatile in price or volume, the bid/ask spread expands. As the

[4] The unit time interval is most appropriately conceived as a single trading day. At the close of trade, financing costs are incurred on the market maker's net position. The market maker adjusts the spread during the trading day to attempt to optimize his ability to cover costs on the closing inventory position.

market-maker's exposure—embodied in the size of his inventory—expands, so does the bid/ask spread.

For the case of risk neutrality, we consider the case where the cost of financing the inventory is taken to be linear in its absolute value.

Case 2. (linear absolute value cost) $C_{t+1} = a|P_{t+1}N_{t+1}|$. Then

$$E_t[C_{t+1}] = aE_t[P_{t+1}]E_t[|N_{t+1}|] = aP_tE_t[|N_{t+1}|].$$

However, now we need to know the distribution functions of buy and sell volumes to calculate the mean of $|N_{t+1}|$:

$$E_t[|N_{t+1}|] = E_t[|N_t - V^{1}_{t+1} + V^{s}_{t+1}|]$$
$$= \int_{N_t - V^{1}_{t+1} + V^{s}_{t+1} \geq 0} (N_t - V^{1}_{t+1} + V^{s}_{t+1})\, \mathrm{d}v^{1}\, \mathrm{d}v^{s} - \int_{N_t - V^{1}_{t+1} + V^{s}_{t+1} < 0} (N_t - V^{1}_{t+1} + V^{s}_{t+1})\, \mathrm{d}v^{1}\, \mathrm{d}v^{s}$$

under the condition that buy and sell volumes are independent and identically distributed. Further, assume they both follow uniform distributions. That is, $f(v) = 1/2\sigma_v$ in the range of $V_0 - \sigma_v \leq v \leq V_0 + \sigma_v$, and zero otherwise.[5]

Then the relationship between $E_t[|N_{t+1}|]$ and $|N_t|$ is nonlinear when the value of $|N_t|$ is relatively small. However, for large enough $|N_t|$ (when $|N_t| \geq V_0 + \sigma_v$), it can be demonstrated (proof available from the authors) that $E_t[|N_{t+1}|] = |N_t|$.[6] That is, $S^*_t = a|N_t|/2V_0$.

In the linear absolute value cost case we find that the bid/ask spread varies directly with the extent of the market-maker's security inventory exposure to market risk. The bigger the market-maker's position (in absolute value), the bigger is the spread.[7]

While some of the empirical implications of this model are indistinguishable from other existing models,[8] such as that the bid/ask spread varies with the price volatility of the underlying security, other predictions are more novel. For example, this model suggests that even temporary imbalances in order flow are likely to lead to larger bid/ask spreads as they tend to increase both net security inventory positions of market-makers and the variance of order flow.

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[5]The mean of the distribution is V_0 and the variance is $\sigma_v^2/3$.

[6]Our setting thus leads to the conclusion that when the initial inventory position is large, the time series of inventory position has a unit root. In contrast, the typical models that allow market-maker to set bid and ask prices separately tend to predict inventory position is mean reverting. Empirical evidence provides indirect support that, at least for high volume stocks, inventory positions may be highly persistent (Hasbrouck and Sofianos, 1993).

[7]Allowing a correlation between volume and price distributions would strengthen the positive relation between the absolute inventory position and bid/ask spread. A sudden positive (negative) surge of inventory should signal negative (positive) price changes are more likely, which makes the existing inventory even more costly to the market maker.

[8]See the references section for a representative list of relevant papers (Amihud and Mendelson, 1980; Copeland and Galai, 1983; Easley and O'Hara, 1987; Garman, 1976; Glosten and Milgrom, 1985; Ho and Stoll, 1981; Madhavan, 2000; O'Hara and Oldfield, 1986; Stoll, 1978).

5. Conclusion

A market-maker's expected average costs increase with the size (absolute value) of his inventory position; the cost of financing inventory increases with its size. Further, in the case where the market-maker faces increasing marginal financing cost, expected average costs increase with security price and order flow volatility. These costs, in equilibrium, must be covered out of the bid/ask spread. Consequently, the bid/ask spread varies directly with security price and order flow volatility and with the size of the market-maker's security inventory position. Since an increasingly turbulent asset market is characterized by imbalance of public buy and sell orders and by increasing price volatility, the model predicts and explains that such a market is likely to be accompanied by deterioration in market liquidity.

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