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sequence economies

A ‘sequence economy’ is a general equilibrium model including markets at a sequence of dates, reopening over time. It is alternative to the Arrow–Debreu model with a full set of futures markets where all exchanges for current and future goods are transacted without transaction cost at a single market date. Sequence economy markets reopen and may be incomplete (some markets, particularly futures, may be inactive) because of transaction costs. The model can provide a microeconomic general equilibrium foundation for the store-of-value function of money, since markets reopening over time create an incentive to carry money and debt intertemporally.

A ‘sequence economy’ is a general equilibrium model in discrete time including specific provision for the availability of markets at a sequence of dates (Hicks, 1939; Radner, 1972). Markets reopen over time, and at each date firms and households act so that plans and prospects for actions on markets available in the future significantly affect their current actions.

This model is in contrast to the Arrow–Debreu model with a (complete) full set of futures markets (Debreu, 1959). There, all exchanges for current and future goods (including contingent commodities, futures contracts contingent on the realization of uncertain events) are transacted on a market at a single point in time. In the Arrow–Debreu model, there is no need for markets to reopen in the future; economic activity in the future consists simply of the execution of the contracted plans. The Arrow–Debreu model with a full set of futures markets appears unsatisfactory in that it denies commonplace observation: futures markets for goods and Arrow securities (contingent contracts payable in money) are not generally available for most dates or a sufficiently varied array of uncertain events; markets do reopen over time. The sequence economy model is an alternative that allows formalization and explanation of these observations.

Several major classes of theoretical model are set in the sequence economy framework: overlapping generations (Balasko and Shell, 1980; 1981; Geanakoplos and Polemarchakis, 1991; Wallace, 1980); temporary equilibrium (Grandmont, 1977; Lucas, 1978), sunspot equilibrium (Chiappori and Guesnerie, 1991), incomplete markets (Geanakoplos, 1990; Magill and Quinzii, 1996). These are general equilibrium models over sequential time emphasizing monetary and financial structure. Each of these areas has a large literature of its own. Typically these models assume a given (incomplete) structure of active financial markets without a detailed foundation for how markets come to be incomplete. This contrasts with the statement of the sequence economy model presented below, which derives market activity and incompleteness endogenously as an equilibrium outcome reflecting transaction costs.

The sequence economy model is particularly suitable to provide a microeconomic foundation for the store-of-value function of money (Hahn, 1971; Starrett, 1973). It is precisely because markets reopen over time that agents may find it desirable to carry abstract purchasing power from one date to succeeding dates. Typically, this will take the form of transactions on spot markets at a succession of dates with money or other financial assets held over time to reflect the (net) excess value of prior sales over purchases. This may occur simply because the model does not provide for futures markets or because futures markets, though available in principle, are in practice inactive. Endogenously determined inactivity of futures markets is the result of transaction costs which tend to make the use of futures markets disproportionately costly compared with spot markets.

There are three principal reasons for the excess cost of futures markets:

1. The necessarily greater complexity of futures contracts may require use of more resources (for example, for record keeping or enforcement) than spot markets.
2. The transaction costs of a futures contract are incurred (partly) at the transaction date, those of an equivalent spot transaction are incurred in the future. The present discounted value of the spot transaction costs incurred in the distant future may be lower than the futures market transaction cost incurred in the present, simply because of time-discounting.
3. Use of a full set of futures markets under uncertainty implies that most contracts transacted become otiose and are left unfulfilled as their effective dates pass and the events on which they were contingent do not occur. There is a corresponding saving in transaction costs associated with reducing the number of transactions required by use of a single spot transaction instead of many contingent commodity contracts, though this reduction may imply a different and inferior allocation of risk-bearing.

We now present a formal pure exchange sequence economy model with transaction costs (Kurz, 1974; Heller and Starr, 1976).

Commodity i for delivery at date τ may be bought spot at date τ or futures at any date t, $1 \leq t < \tau$. The complete system of spot and futures markets is available at each date (although some markets may be inactive). The time horizon is date K; each of H households is alive at time 1 and cares nothing about consumption after K. There are n commodities deliverable at each date; in the monetary interpretation of the model spot money is one of the goods. At each date and for each commodity, the household has available the current spot market, and futures markets for deliveries at all future dates. Spot and futures markets will also be available at dates in the future and prices on the markets taking place in the future are currently known. Thus in making his purchase and sale decisions, the household considers without price uncertainty whether to transact on current markets or to postpone transactions to markets available at future dates. There is a sequence of budget constraints, one for the market at each date. That is, for every date, the household faces a budget constraint on the spot and futures transactions taking place at that date, (4) below. The value of its sales to the market at each date (including delivery of money) must balance its purchases at that date.

In addition to a budget constraint, the agent's actions are restricted by a transaction technology. This technology specifies for each complex of purchases and sales at date t, what resources will be consumed by the process of transaction. It is because transaction costs may differ between spot and futures markets for the same good that we consider the reopening of markets allowed by the sequence economy model. Specific provision for transaction cost is introduced to allow an endogenous determination of the activity or inactivity of markets. In the special case where all transaction costs are nil, the model is unnecessarily complex; there is no need for the reopening of markets, and the equilibrium allocations are identical to those of the Arrow–Debreu model. Conversely, in the case where some futures markets are prohibitively costly to operate and others are costless, then there is an incomplete array of spot and futures markets and the model is an example of that of Radner (1972).

All of the n-dimensional vectors below are restricted to be non-negative.

$x_\tau^h(t)$ = vector of purchases for any purpose at date t by household h for delivery at date τ.
$y_\tau^h(t)$ = vector of sales analogously defined.
$z_\tau^h(t)$ = vector of inputs necessary to transactions undertaken at time t. The index τ again refers to date at which these inputs are actually delivered.
$\omega^h(t)$ = vector of endowments at t for household h.
$s_h(t)$ = vector of goods coming out of storage at date t.
$r_h(t)$ = vector of goods put into storage at date t.
$p_\tau(t)$ = price vector on market at date t for goods deliverable at date τ.

With this notation, $p_{it}(t)$ is the (scalar) spot price of good i at date t, and $p_{i\tau}(t)$ for $\tau > t$ is the futures price (for delivery at τ) of good i at date t.

The (non-negative) consumption vector for household h is

$$c^h(t) = \omega^h(t) + \sum_{\tau=1}^{t} \left[x_t^h(\tau) - y_t^h(\tau) - z_t^h(\tau) \right] + s^h(t) - r^h(t) \geqq 0, \qquad (t = 1, \ldots, K). \tag{1}$$

That is, consumption at date t is the sum of endowments plus all purchases past and present with delivery date t minus all sales for delivery at t minus transaction inputs with date t (including those previously committed) plus what comes out of storage at t minus what goes into storage. We suppose that households care only about consumption and not about which market consumption comes from.

The household is constrained by its transaction technology, $T_h(t)$, and by its storage technology, $S_h(t)$. $T_h(t)$ specifies the resources, for example, how much leisure time and shoeleather, must be used to carry out a transaction. Let $x^h(t)$ denote the vector of $x_\tau^h(t)$'s [and similarly for $y^h(t)$ and $z^h(t)$]. We insist

$$[x^h(t), y^h(t), z(t)] \in T^h(t), \qquad (t = 1, \ldots, K). \tag{2}$$

Naturally, storage input and output vectors must be feasible, so

$$[r^h(t), s^h(t+1)] \in S^h(t), \qquad (t = 1, \ldots, K-1). \tag{3}$$

The budget constraints for household h are then:

$$p(t) \cdot x^h(t) \leqq p(t) \cdot y^h(t), \qquad (t = 1, \ldots, K). \tag{4}$$

Households may transfer purchasing power forward in time by using futures markets and by storage of goods that will be valuable in the future. Purchasing power may be carried backward by using futures markets. But these may be very costly transactions. In a monetary interpretation of the model, where money and promissory notes are present, the household can either hold money as a store of wealth, or it can buy or sell notes.

Let household h's action at date t be denoted $a^h(t) \equiv [x^h(t) y^h(t), z^h(t), r^h(t), s^h(t)]$. Let a^h be a vector of the $a^h(t)$'s, and define x^h, y^h, z^h, r^h and s^h similarly. Define $B^h(p)$ as the set of a^h's which satisfy constraints (1)–(4). The household chooses $a^h(t)$ to maximize $U^h(c^h)$ over $B^h(p)$. Denote the demand correspondence (i.e. the set of maximizing a^h's) by $\gamma^h(p)$.

The model can be interpreted as monetary or non-monetary. We think of money as simply a 0th good that does not enter household preferences.

Futures contracts in money are discounted promissory notes. $x^h_{0t}(t)$ is h's monetary receipts at $t, x^h_{0\tau}(t)$ is h's note purchase at t due at τ. Money is not treated as numeraire – positivity of its value cannot be assumed – it has a price $p_{0t}(t)$.

The correspondences $\gamma^h_t(p)$ are always homogeneous of degree zero in $p(t)$, as is seen from the definition of $B^h(p)$. We can therefore restrict the price space to the simplex. Let S^t denote the unit simplex of dimensionality, $n(K-t+1)$. Let $P = X^K_{t=1}S^t$, where X denotes a Cartesian product.

An equilibrium of the economy is a price vector $p^* \in P$ and an allocation a^{h^*}, for each h, so that $a^{h^*} \in \gamma^h(p^*)$ for all h and

$$\sum_{h=1}^{H} x^{h^*} \leqq \sum_{h=1}^{H} y^{h^*} \tag{5}$$

(the inequality holds coordinate-wise), where for any good i, t, τ such that the strict inequality holds in (5) it follows that $p^*_{i\tau}(t) = 0$. The equilibrium of a monetary economy is said to be *non-trivial* (that is, the economy is really monetary) if $p^*_{0t}(t) \neq 0$ for all t. Sufficient conditions for existence of equilibrium are continuity and convexity requirements typical of an Arrow–Debreu model appropriately extended. Transaction costs are often thought to be non-convex, leading to approximate equilibrium rather than full equilibrium results (Heller and Starr, 1976).

In the case of fiat (unbacked) money, existence of non-trivial monetary equilibrium requires additional structure designed to maintain positivity of the price of money (boundedness of the price level expressed in monetary terms). This may take a variety of forms: the model may arbitrarily require that fiat money be held or turned in at a finite horizon; households may expect fiat money to be valuable in the future sustaining its value in the present; there may be taxes payable in fiat money. Alternatively, the model may assume an infinite horizon (typical of the overlapping generations model) so that the lack of backing for fiat money need not be experienced (though a nil value of fiat money in equilibrium is still a logical possibility).

In contrast to the Arrow–Debreu economy, a sequence economy equilibrium allocation is not generally Pareto efficient. This is not due simply to the presence of transaction costs; transaction costs technically necessary to a reallocation must be incurred, and they represent no inefficiency. The Arrow–Debreu model, however, uses a lifetime budget constraint. The corresponding constraint here is the sequence of budget constraints in (4). Transfer of purchasing power intertemporally – costless in the Arrow–Debreu model – is here a resource using activity; it requires purchase and sale of assets with resultant transaction cost. But the intertemporal transfer of purchasing power, unlike reallocation of goods among households, is needed not to satisfy technical or consumption requirements but rather to satisfy the administrative requirements of sequential budget constraint embodied in (4). Hence technically feasible Pareto-improving reallocations may be prevented in equilibrium by prohibitive transaction costs which would have to be incurred to satisfy the purely administrative requirements of crediting and debiting agents' budgets intertemporally (Hahn, 1971). If trade in monetary instruments is costless, however, then an equilibrium allocation is Pareto efficient (Starrett, 1973). Thus the sequence economy model provides a value-theoretic foundation for the store-of-value role of money.

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See also

<xref=xyyyyyy> general equilibrium.

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Index terms

Arrow–Debreu model
budget constraint
fiat money
futures markets
general equilibrium
incomplete markets
infinite horizons
intertemporal transfers
overlapping generations models
sequence economies
spot markets
store-of-value function of money
sunspot equilibrium

temporary equilibrium
transaction costs