Liquidity Creates Money and Debt: An Intertemporal Linear Trading Post Model

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Abstract

In a pure exchange trading post economy over time linear transaction costs are recouped through a bid-ask price spread. There is a full set of spot and futures markets at each date, trivially generating rational price expectations. For every two commodities, there is a trading post where they can be traded pairwise for one another: spot for spot, spot for future, future for future. Most trading posts will be priced but inactive in equilibrium. One commodity (denoted $\boldsymbol{m}$) has, by assumption, distinctly low spot and futures transaction cost. Spot markets are assumed to have lower transaction cost than futures markets. Bid and ask prices for all goods, spot, in exchange for the low-transaction-cost good $\boldsymbol{m}$ reflect its low transaction cost, creating a narrow bid-ask spread. In general equilibrium and assuming the absence of double coincidence of wants, the low-transaction-cost commodity $\boldsymbol{m}$ becomes the common medium of exchange. Further, storing $\boldsymbol{m}$ or engaging in futures market transactions for $\boldsymbol{m}$ (borrowing and lending) make $\boldsymbol{m}$ and its debt instruments ($\boldsymbol{m}$-futures) the prevailing intertemporal store of value. The rate of interest on $\boldsymbol{m}$ is implied in the pricing of $\boldsymbol{m}$-futures. $\boldsymbol{m}$'s specialization as the common medium of exchange and store of value is the result of decentralized exchange and competitive pricing. There is no role for governmental or legal tender designation, or consensus among transactors. Monetization of exchange using $\boldsymbol{m}$ as 'money' is fully decentralized by general equilibrium pricing.

A salute to Roy Radner

Roy Radner has been a colleague and a friend for half a lifetime. In the late 1960s we participated in the biweekly Berkeley-Stanford seminar on mathematical economics. Those were heady days; the field was young and new discoveries appeared rapidly. It was there that I first saw Roy's landmark paper "Competitive Equilibrium under Uncertainty". At one seminar meeting, a colleague remarked to me: "What a useless visit to Berkeley this was! I didn't get a chance to talk with Roy!" Roy was a leader then, as he is now.

In the following summers we attended the Stanford summer IMSSS seminar. Those were intense several-week sessions with superb minds from across the globe.

In the course of a career Roy's research has spanned broad areas of economic theory. My favorite is his analysis of the economics of uncertainty. Roy knows a lot about uncertainty — whereas the rest of us are uncertain about a lot.

Roy's dedication to precision and insight in his research is steadfast. Roy's long and continuing excellence in teaching and research is an inspiring example.

All of us here are pleased to salute an outstanding leader of our field.

1 Money in General Equilibrium Theory

For half a century the general equilibrium model, in its Arrow-Debreu form, has been the touchstone of economic analysis. Prof. Hugo Sonnenschein remarked [1]:

> The Arrow-Debreu model, as communicated in *Theory of Value* changed basic thinking, and it quickly became the standard model of price theory. It is the 'benchmark' model in Finance, International Trade, Public Finance, Transportation, and even macroeconomics. ... In rather short order it was no longer 'as it is' in Marshall, Hicks, and Samuelson; rather it became 'as it is' in *Theory of Value*.

But there is a problem. The model has no place for a theory of money. Hahn (1982) poses the problem for price theory in the following way:

> The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best developed model is, of course, the Arrow-Debreu version of a Walrasian general equilibrium. A first, and...difficult...task is to find an alternative construction without...sacrificing the clarity and logical coherence ... of Arrow-Debreu.

Encompassing monetary structure in the Arrow-Debreu model is a difficult problem. But the source of the problem is elementary. The Arrow-Debreu model includes a single meeting of a market where all transactions take place, profits are earned and distributed, sales and purchases undertaken, and budgets balanced. Firms and households make only one grand multi-dimensional transaction. With only a single transaction there is no room for money or debt, a carrier of value between transactions. This paper continues the investigation of a class of models, the trading

[1] Prof. Sonnenschein's remarks at the UC Berkeley conference in honor of the memory of Gerard Debreu in 2005, quoted in Starr (2011).

post model, that decomposes the trading behavior of households into many separate transactions, each of which fulfills a budget constraint, Starr (2012). Walras (1874) forms the picture (assuming m distinct commodities):

> "we shall imagine that the place which serves as a market for the exchange of all the commodities (A), (B), (C), (D) ... for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have $\frac{m(m-1)}{2}$ special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their ... rates of exchange..."[2]

The role of a carrier of value between transactions naturally, endogenously, arises there. The focus in this paper is to extend the model over time, so that debt, interest on money, and price expectations, can enter the analysis.

Jevons (1875) notes that in the rare instance of two traders having mutually complementary demands and supplies — "double coincidence of wants" — then barter trade can proceed successfully. The array of unsatisfied supplies and demands for good i and for good j would then include one trader with an excess supply of i and an excess demand for j, and a mirror-image second trader with the opposite unsatisfied supply and demand. That is the rare event where traders can directly, without an intermediary good, arrange pairwise mutually improving trades. The class of examples developed in this essay does not admit this complementary mix of demands and supplies.

[2] Shapley and Shubik (1977) and Starr (2003) also treat the trading post model. See also Banerjee and Maskin (1996) and Howitt (2005).

1.1 The Most Saleable Good

The most elementary function of money — the medium of exchange — is as a carrier of value held between successive transactions. Carl Menger (1892) reminds us that the distinguishing feature of the medium of exchange should be liquidity:

> why...is...economic man ...ready to accept a certain kind of commodity, *even if he does not need it*, ... in exchange for all the goods he has brought to market[?]... *The theory of money necessarily presupposes a theory of the saleableness [Absätzfahigkeit] of goods ...* [Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution... Men ... exchange goods ... for other goods ... more saleable....[which] become generally acceptable media of exchange.
>
> Money has not been generated by law. In its origin it is a social, and not a state-institution. [emphasis in original] [3]

"Saleableness" is liquidity. Though Menger notes many dimensions to liquidity (delay, uncertainty, search, ...), a simple characterization is the difference between the bid price and the ask price. A commodity that acts as a medium of exchange is necessarily repeatedly bought (accepted in trade) and sold (delivered in trade). Therefore a good with a narrow spread between bid and ask price is priced to encourage households to use it as a carrier of value between trades, as a medium of exchange with relatively low cost.

This paper posits a trading post model. For any two commodities — including goods distinguished by date of delivery — there is a submarket where the two goods

[3] See Radford (1945) on the evolution of a cigarette currency and Newhouse (2004) on convergence to monetary equilibrium in a 3-commodity model. Banerjee and Maskin (1996) focus on the ease or difficulty of assessing quality — a form of saleableness — as the rationale for a common medium of exchange. Roscher (1878) reiterates the notion of money as the most 'saleable' good.

can be exchanged for one another. All trade takes place in this pairwise form. Prices are endogenously determined by market-clearing. Transaction costs at the trading posts are communicated to trading households (and recouped by the trading posts) by a spread between bid and ask prices at the trading post. The pattern of trade across trading posts is determined endogenously. A *barter* equilibrium occurs when most trading posts are active in equilibrium, one for each pair of distinct goods. Conversely, if most trading posts are inactive in equilibrium, and there is active trade concentrating on the small number of posts trading a single good pairwise against all others, then the equilibrium will be described as *monetary*, with the single commonly traded good as *commodity money*.

Figure 1 depicts the distinction between barter and monetary trade. Each node represents a commodity; each chord represents an active market of trade in the commodity pair. The right hand side depicts barter trade with all goods trading actively for one another. The left hand side depicts monetary trade with all trade going through good $\boldsymbol{1}$ as the common medium of exchange. Figure 2 depicts monetary trade over time. At each date all trade in goods of that date goes through good $\boldsymbol{m}$ as the common medium of exchange. Trade over time goes through the money markets: good $\boldsymbol{m}$ deliverable at differing dates traded for one another. There is no direct trade of goods other than $\boldsymbol{m}$ deliverable at different dates.

2 Trading Posts and *Quid Pro Quo*

The model of this essay will assume N commodities available at T dates. The notation $i(\sigma)$ represents good i available at date σ. There is a full set of futures markets at each date, so for each $t = 1, 2, \ldots, T$ there is the opportunity to trade each $i(\sigma)$ where $\sigma \geq t$.

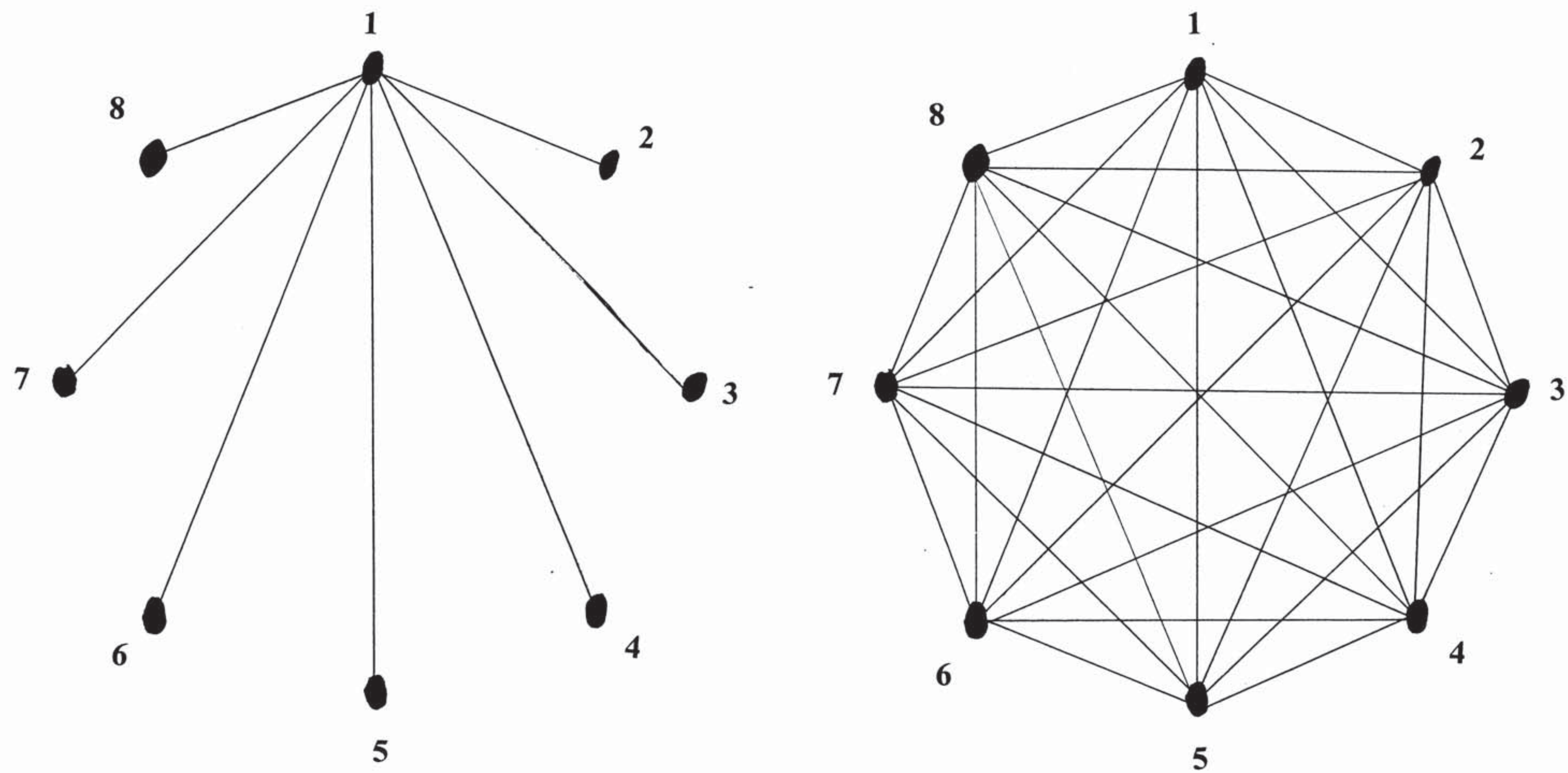

Figure 1

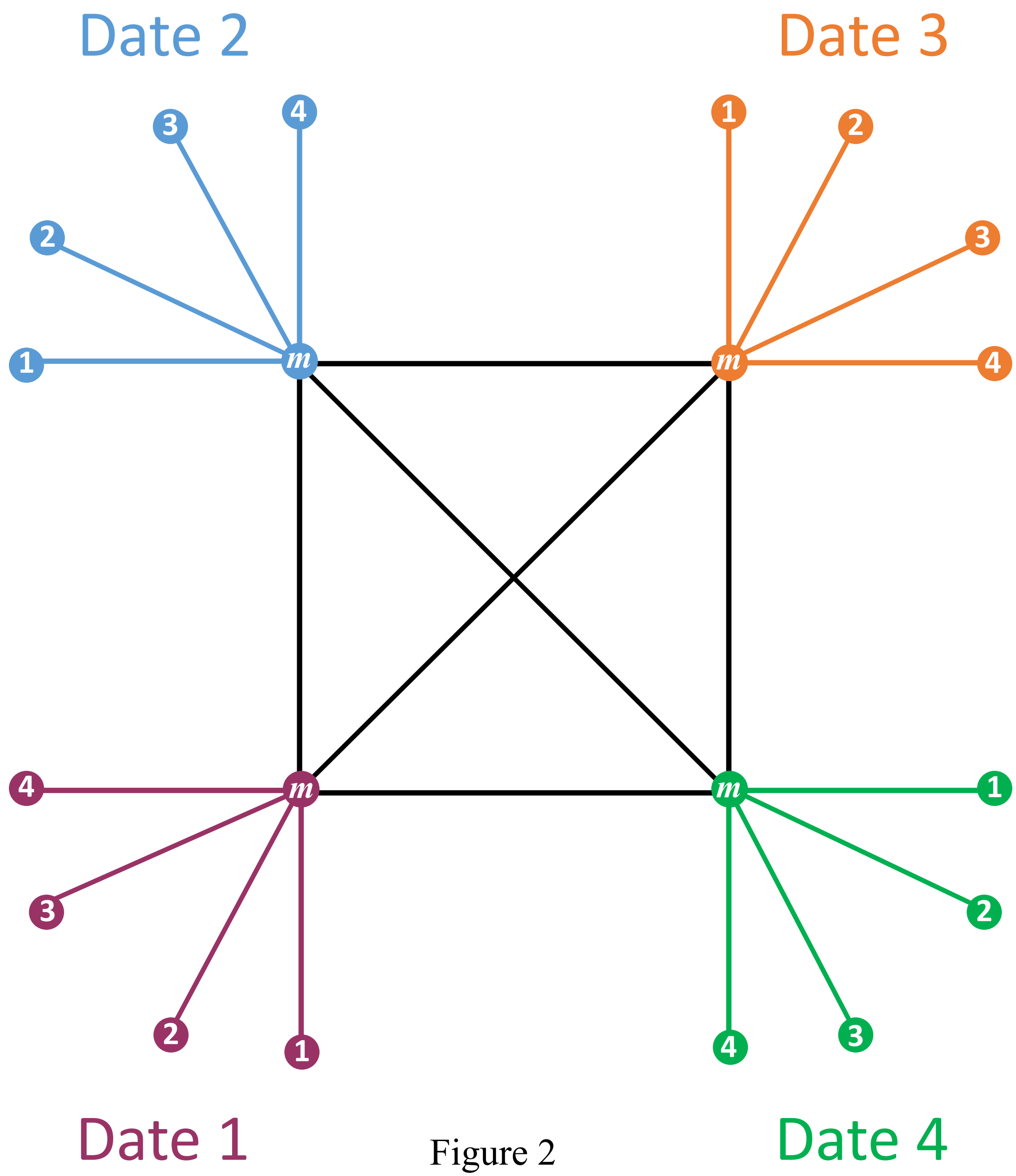

Figure 2

Goods are are traded in pairs — good $i(\sigma)$ for good j(τ) — at specialized trading posts. Trading posts are available at each date t = 1, 2, . . . , T, for $t \leq \sigma, \tau$. The trading post for trade of good $i(\sigma)$ versus good $j(\tau)$ (and vice versa) meeting at date t is designated $\{i(\sigma), j(\tau), t\}$; trading post $\{i(\sigma), j(\tau), t\}$ is the same trading post as $\{j(\tau), i(\sigma), t\}$. The trading post is a business firm, the market maker in trade between goods $i(\sigma)$ and $j(\tau)$. The trading post actively buys and (re)sells both $i(\sigma)$ and $j(\tau)$. Trade as a resource using activity is modeled by describing the post's transaction costs. The notion of transaction cost summarizes costs that in an actual economy are incurred by retailers, wholesalers, individual firms and households. The bid/ask spread summarizes these costs to the model's transactors.

The trading post $\{i(\sigma), j(\tau), t\}$ defrays the transaction cost $C^{\{j(\tau),i(\sigma),t\}}$ through the retained $i(\sigma)$ and $j(\tau)$ left with the post through the difference between the bid and ask prices. A very general transaction cost function $C^{\{j(\tau),i(\sigma),t\}}$ would distinguish transaction costs differing among commodities, including differences in durability, portability, recognizibility, divisibility. The transaction cost structure posited here, on the contrary, is linear and almost completely uniform. It is surely oversimplified: transaction costs are assessed only in the goods transacted. This simplifies the accounting for cost. The usage ignores that transaction costs are incurred in labor, capital, additional resources. In addition, this simplification ignores the timing of inputs to transaction costs at date t; one might reasonably expect that they would also be dated t; the (purposefully) oversimplified usage here is that inputs are incurred in the dated goods traded, not in the goods dated at the time of trade. This is an unfortunate usage but a useful simplification.

The population of households is denoted Θ, consisting of a mix of subpopulations (with different tastes and endowments). Jevons reminds us that the mix of household tastes is essential to the discussion of media of exchange. For example, if the endowment allocation is Pareto efficient, then there will be no exchange in equilibrium and no medium of exchange. Conversely, Jevons insists that if the endowment allocation displays absence of double coincidence of wants, then indirect trade and use of a medium of exchange is likely to result.

Households formulate their trading plans deciding how much of each good to trade at each pairwise trading post. This leads to the rather messy notation:

$b_\ell^{h,\{i(\sigma),j(\tau),t\}}$ = planned purchase of good ℓ by household h at trading post $\{i(\sigma), j(\tau), t\}$.

$s_\ell^{h,\{i(\sigma),j(\tau),t\}}$ = planned sale of good ℓ by household h at trading post $\{i(\sigma), j(\tau), t\}$

There is some redundant generality in this notation, since the only goods actually traded at $\{i(\sigma), j(\tau), t\}$ will be $i(\sigma)$ and $j(\tau)$.

The bid prices (the prices at which the trading post will buy from households) at $\{i(\sigma), j(\tau), t\}$ are $q_i^{\{i(\sigma),j(\tau),t\}}$, $q_j^{\{i(\sigma),j(\tau),t\}}$ for goods $i(\sigma)$ and $j(\tau)$ respectively. The price of $i(\sigma)$ is in units of $j(\tau)$. The price of $j(\tau)$ is in units of $i(\sigma)$. The ask price (the price at which the trading post will sell to households) of $j(\tau)$ is the inverse of the bid price of $i(\sigma)$ (and vice versa). That is, $(q_i^{\{j(\tau),i(\sigma),t\}})^{-1}$ and $(q_j^{\{j(\tau),i(\sigma),t\}})^{-1}$ are respectively the ask prices of $j(\tau)$ and $i(\sigma)$ at $\{i(\sigma), j(\tau), t\}$. The trading post $\{i(\sigma), j(\tau), t\}$ covers its costs by the difference between the bid and ask prices of $j(\tau)$ and $i(\sigma)$, that is, by the spread $(q_j^{\{j(\tau),i(\sigma),t\}})^{-1} - q_i^{\{j(\tau),i(\sigma),t\}}$ and the spread $(q_i^{\{j(\tau),i(\sigma),t\}})^{-1} - q_j^{\{j(\tau),i(\sigma),t\}}$. Transaction costs at the trading post defrayed through goods $j(\tau)$ and $i(\sigma)$, acquired in trade through the difference in bid and ask prices.

3 Transaction Costs

Consider trading posts with a linear transaction cost structure. The trading post buys goods from households and resells them or retains them to cover transaction costs. Trading futures contracts is more costly than spot contracts, and trading good m is low in cost. Let $1 > \epsilon > \delta > 0$. Let the cost structure of trading post $\{i(\sigma), j(\tau), t\}$, $i, j = 1, 2, ..., N; \sigma, \tau = 1, 2, \ldots, T; i(\sigma) \neq j(\tau), i(\sigma) \neq m \neq j(\tau); t \neq \sigma$ or $t \neq \tau$, be:

$C^{\{i(\sigma),j(\tau),t\}} = \epsilon \times$ (volume of goods $i(\sigma)$ and $j(\tau)$ purchased by the post)

Marginal cost of trading $i(\sigma)$ for $j(\tau)$ futures is ϵ times the gross quantity traded, where $1 > \epsilon > 0$. The trading post expects to cover its transaction costs through the bid/ask spread.

For the case $t = \tau, \sigma$, trading is spot and occurs at lower transaction cost than futures transactions.

$C^{\{i(t),j(t),t\}} = \delta \times$ (volume of goods $i(t)$ and $j(t)$ purchased by the post)

Trading good ***m(t)*** is assumed to be costless. This is where the fix goes in — ***m(t)***, good m spot, is being set up as the natural money. We'll see how that works out in equilibrium. Thus, for $t \neq \tau$,

$C^{\{m(t),j(\tau),t\}} = \epsilon \times$ (volume of good $j(\tau)$ purchased by the post) ,

for $j = 1, 2, \ldots, m-1, m+1, \ldots, N$. Spot transactions including spot ***m(t)*** are particularly low transaction cost.

$C^{\{m(t),j(t),t\}} = \delta \times$ (volume of good $j(t)$ purchased by the post) ,

for $j = 1, 2, \ldots, m-1, m+1, \ldots, N$.

Finally, futures market transactions involving ***m*** and itself are of peculiarly low cost

$C^{\{m(\sigma),m(\tau),t\}} = 0$, $\sigma, \tau \geq t$.

In an economy of N commodities and T time periods there are $[\frac{1}{2}N(N-1)] \times [\frac{1}{2}T(T-1)]$ trading posts each with two posted prices (bid for one good in terms of a second, and bid price of the second in units of the first) totaling $[\frac{1}{2}N(N-1)] \times [T(T-1)]$ pairwise price ratios. Prices are posted at all trading posts — including those without active trade.

The market equilibrium guided by the price system here must answer the question: which trading posts operate at positive trading volume? In actual economies, most conceivable pairwise commodity trades do not occur. A trading post becomes unattractive in equilibrium, and will have zero trading volume (a corner solution), when its bid/ask spread is wide enough to discourage trade.

4 Households

Let $[i(\sigma)$,$j(\tau)]$ denote a household endowed with good $i(\sigma)$ who prefers good $j(\tau)$; $i(\sigma) \neq j(\tau), i, j = 1, 2, ..., N; 1 \leq \sigma, \tau \leq T$. Households are assumed to have a time preference factor ρ. The algebra of this model is most convenient when each household's endowment has the same present value. Thus, household $[i(\sigma)$,$j(\tau)]$'s endowment is ρ^σ unit of commodity $i(\sigma)$. Denote the endowment of $[i(\sigma)$,$j(\tau)]$ as $r_i^{[i(\sigma),j(\tau)]} = \rho^\sigma$. $[i(\sigma)$,$j(\tau)]$'s utility function is $u^{[i(\sigma),j(\tau)]}(x_{1(1)}, x_{2(1)}, ..., x_{k(\nu)}, ..., x_{N(T)}) = \rho x_{j(\tau-1)} + x_{j(\tau)} + \frac{1}{\rho} x_{j(\tau+1)}$.[4] That is, household $[i(\sigma)$,$j(\tau)]$ values goods j$(\tau-1)$, $j(\tau)$, j$(\tau+1)$ only, with a time preference factor ρ. He cares for $i(\sigma)$ only as a resource to trade for $j(\tau-1)$, $j(\tau)$, $j(\tau+1)$. This is obviously an immense oversimplification — but it serves to focus the issue.

Consider a population denoted Θ of households displaying a complete absence of double coincidence of wants. Ω denotes the greatest integer $\leq (N-1)/2$. There are $N \times \Omega \times T^2$ households each endowed with a dated commodity and each desiring a distinct commodity-dated good different from its endowment. There are $\Omega \times T$ households endowed with good $1(\sigma)$, each preferring a distinct choice of goods 2, 3, 4, ..., $\Omega + 1$ at the full range of alternative dates, $\tau = 1, 2, \cdots, T$. Thus households endowed with $1(\sigma)$ are arrayed with desires : [1(σ),2(1)],[1(σ),2(2)],$\cdots$, [1(σ),2(T)], [1(σ),3(1)],. . ., [1(σ),$\Omega+1(T)$]. There are $\Omega \times T$ households endowed with good $2(\sigma)$,

[4] Of course, this notation applies only where $1 \leq \tau - 1, \tau, \tau + 1 \leq T$. Where the inequality is not fulfilled, we delete the good with the nonconforming date from the utility function.

preferring respectively goods 3, 4, 5, ..., Ω+2, over the full range of available dates 1, 2, ..., T. They are [2(σ),3(1)],[2(σ),3(2)], ..., [2(σ),3(T)], [2(σ),4(1)],$\cdots$, [2(σ),$\Omega+2(T)$]. The roll call of households proceeds so forth, through [N(T), 1(1)],..., [N(T), 2(1)], ..., [N(T),$\Omega(T)$].

One way to think of Θ is that its elements [$i(\sigma)$,$j(\tau)$] are set round a clock-face at a position corresponding to the endowed good, $i(\sigma)$, eager to acquire $j(\tau)$. $j(\tau)$ being located clockwise from $i(\sigma)$.

In the model developed here, reflecting complete absence of double coincidence of wants, for each household endowed with good $i(\sigma)$ and desiring good $j(\tau)$, [$i(\sigma)$,$j(\tau)$], there is no precise mirror image, [$j(\tau)$,$i(\sigma)$]. Nevertheless, there are $\Omega \times T$ households endowed with ρ^σ unit of commodity $1(\sigma)$, and $\Omega \times T$ households strongly preferring commodity $1(\sigma)$ to all others. That is true for each good, deliverable at each date. Thus gross supplies equal gross demands, though there is no immediate opportunity for any two households to make a mutually advantageous trade. Jevons (1875) tells us that this is precisely the setting where money is suitable to facilitate trade. Population Θ displays absence of "double coincidence of wants."

4.1 Household transactions, consumption, and holdings

A typical household $h \in \Theta$, has an initial endowment $r^h(1) \in R_+^{NT}$; $r_n^h(1)$ is h's endowment of good n. The character '1' here denotes the date in trading time of h's ownership of $r^h(1)$.

Household h at date t makes sales of good n, $s_n^h(t) = \sum_{\{n,i(\sigma),t\}} s_n^{h\{n,i(\sigma),t\}}$. The summation here is over the trading posts at date t trading good n for spot or future goods $i(\sigma)$. $s^h(t) \in R^{NT}$ is the vector $(s^h_{1(1)}(t), s^h_{1(2)}(t), \ldots, s^h_{1(T)}(t), \ldots, s^h_n(t), \ldots, s^h_{N(1)}(t), \ldots, s^h_{N(T)}(t))$.

Household h at date t makes purchases of good n, $b^h_n(t) = \sum_{\{n,i(\sigma),t\}} b^{h\{n,i(\sigma),t\}}_n$. $b^h(t) \in R^{NT}$ is the vector $(b^h_{1(1)}(t), b^h_{1(2)}(t), \ldots, b^h_{1(T)}(t), \ldots, b^h_n(t), \ldots, b^h_{N(1)}(t), \ldots, b^h_{N(T)}(t))$.

Household h at date t consumes some of good n, $c^h_n(t)$ for $n = i(t), i = 1, 2, \ldots, N$. $c^h(t) \in R^{NT}$ is the vector $(c^h_{1(1)}(t), c^h_{1(2)}(t), \ldots, c^h_{1(T)}(t), \ldots, c^h_{N(1)}(t), \ldots, c^h_n(t), \ldots c^h_{N(T)}(t))$. Generally, we'd expect $c^h_{i(\kappa)}(t) = 0$ for $\kappa \neq t$.

In addition, h's holdings of goods in inventory are subject to depreciation, $d^h_n(t)$ for $n = i(t), i = 1, 2, \ldots, N$. In the monetary equilibrium developed below, traders avoid depreciation by holding no goods — except futures contracts, assumed non-depreciable — in inventory.

h's holdings coming into date $t > 1$, denoted $r^h(t)$, are

$r^h(t) \equiv r^h(t-1) - d^h(t-1) + b^h(t-1) - s^h(t-1) - c^h(t-1)$. All of the arguments in this expression are vectors in R^{NT}.

5 Trading Post Balance Constraints

Given $q_i^{\{j(\tau),i(\sigma),t\}}, q_j^{\{j(\tau),i(\sigma),t\}}$, for all $\{i(\sigma), j(\tau), t\}$ so that $\sigma, \tau \geq t$, household h then forms its buying and selling plans, in particular deciding which trading posts to use to execute his desired trades. Household $h \in \Theta$ faces the following constraints on its transaction plans.

Trading Post Balance Constraints:

(T.i) $b_n^{h\{j(\tau),i(\sigma),t\}} > 0$ only if $n = i(\sigma), j(\tau)$; $s_n^{h\{j(\tau),i(\sigma),t\}} > 0$ only if $n = i(\sigma), j(\tau)$.

(T.ii) $b_i^{h\{j(\tau),i(\sigma),t\}} \leq q_j^{\{j(\tau),i(\sigma),t\}} \cdot s_j^{h\{j(\tau),i(\sigma),t\}}$, $b_j^{h\{j(\tau),i(\sigma),t\}} \leq q_i^{\{j(\tau),i(\sigma),t\}} \cdot s_i^{h\{j(\tau),i(\sigma),t\}}$ for each $n = i(\sigma), j(\tau)$.

(T.iii) $r^h(T+1) \equiv r^h(T) - d^h(T) + b^h(T) - s^h(T) - c^h(T) \geq 0$, where 0 is the zero vector in R^{NT} and the inequality holds co-ordinatewise.

Note that condition (T.ii) defines a budget balance requirement at the transaction level, implying the decentralized character of trade. Since the budget constraint applies to each pairwise transaction separately, there may be a demand for a carrier of value to move purchasing power between distinct transactions. h faces the array of bid prices $q_i^{\{j(\tau),i(\sigma),t\}}, q_j^{\{j(\tau),i(\sigma),t\}}$ and chooses $s_n^{h\{j(\tau),i(\sigma),t\}}$ and $b_n^{h\{j(\tau),i(\sigma),t\}}$, $n = i(\sigma), j(\tau); i(\sigma) \neq j(\tau)$, t = 1, 2, ..., T, to maximize $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii). That is, h chooses which pairwise markets to transact in and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints.

Budgets must balance at each trading post — that is, you pay for what you get not only over the course of all trade (as in the Arrow-Debreu model) but at each trading post separately. A household delivers good $i(\sigma)$ to trading post $\{i(\sigma), j(\tau), t\}$ and the delivery is evaluated at the post's bid price determining how much good $j(\tau)$ the household receives. Budget balance requires that the values be equal. Then the budget constraint facing household h at $\{i(\sigma), j(\tau), t\}$ is $b_i^{h\{i(\sigma),j(\tau),t\}} \leq s_j^{h\{i(\sigma),j(\tau),t\}} \cdot q_j^{\{i(\sigma),j(\tau),t\}}$.

Household h's net acquisition of good $i(\sigma)$ then at date t is $\sum_{j=1}^{N}\sum_{\tau=t}^{T}[b_i^{h\{i(\sigma),j(\tau),t\}} - s_i^{h\{i(\sigma),j(\tau),t\}}]$.

(T.iii) is a terminal nonnegativity condition. Though households may go into debt (either promising to deliver goods in future or promising to deliver $\boldsymbol{m}$ at a future date) throughout the course of trade, their holdings must be nonnegative at the conclusion of trade, at termination date T.

6 Marginal Cost Prices

Denote the bid price of good $i(\sigma)$ at trading post $\{i(\sigma), j(\tau), t\}$ as $q_i^{\{i(\sigma),j(\tau),t\}}$. Recall the following constants:

$\rho =$ utility time preference factor;

$\delta =$ proportional spot market transaction cost on goods other than $\boldsymbol{m}$;

$\epsilon =$ proportional futures market transaction cost for goods other than $\boldsymbol{m}$;

$0 =$ proportional transaction cost on spot and futures markets for good $\boldsymbol{m}$.

Bid prices of $i(\sigma)$ for $j(\tau)$ (and vice versa) will be discounted by their transaction costs. In the monetary equilibrium developed below, with good $\boldsymbol{m}$ performing the monetary function, all of the barter markets are thin, so all of the transaction cost — on the trading posts not trading in m — on both sides of the market are attributed to the bid price of each good. Then marginal cost pricing results in the following bid prices at the spot and futures trading posts for goods $i, j \neq m$:

- At spot trading post $\{i(t), j(t), t\}$, $q_i^{\{i(t),j(t),t\}} = (1-\delta)^2$;

- At a futures trading post where both goods have the same delivery date $\{i(\sigma), j(\sigma), t\}$, $q_i^{\{i(\sigma),j(\sigma),t\}} = (1-\epsilon)^2$;

- At a futures trading post where delivery dates differ,$\sigma \neq \tau$, $\{i(\sigma), j(\tau), t\}$, $q_i^{\{i(\sigma),j(\tau),t\}} = \rho^{(\tau-\sigma)}(1-\epsilon)^2$. Note that $(\tau - \sigma)$ may be positive or negative, reflecting the time value of earlier endowment or the time discount of a later endowment.

- At a trading post where one good is spot and the other future, $\{i(t), j(\tau), t\}$, $q_i^{\{i(t),j(\tau),t\}} = \rho^{(\tau-t)}(1-\delta)(1-\epsilon)$.

At the trading posts where good $\boldsymbol{m}$ is traded, transaction costs are lower and bid prices correspondingly higher.

- At a spot trading post including good $\boldsymbol{m}$, $\{i(t), m(t), t\}$, $q_i^{\{i(t),m(t),t\}} = (1-\delta)$ and $q_m^{\{i(t),m(t),t\}} = 1$.

- Futures transactions including good $\boldsymbol{m}$ alone are priced for time preference but include no transaction cost. At trading post $\{m(\sigma), m(\tau), t\}$, $q_{m(\sigma)}^{\{m(\sigma),m(\tau),t\}} = \rho^{(\tau-\sigma)}$. Note that $(\tau - \sigma)$ may be positive, negative, or zero.

7 Marginal cost pricing equilibrium

The market clearing equilibrium concept is an array of prices $q_i^{o\{i(\sigma),j(\tau),t\}}$ and trades $b_i^{oh\{i(\sigma),j(\tau),t\}}$, $s_j^{oh\{i(\sigma),j(\tau),t\}}$ for $h \in \Theta$. It is said to be a *marginal cost pricing equilibrium* if each household $h \in \Theta$ optimizes utility subject to budget at prevailing prices, each trading post clears, and trading posts cover marginal costs through bid/ask spreads.

More formally, a marginal cost pricing equilibrium under the transaction cost function above consists of $q_i^{o\{i(\sigma),j(\tau),t\}}$, $q_j^{o\{i(\sigma),j(\tau),t\}}$, $1 \leq i, j \leq N, i(\sigma) \neq j(\tau), t = 1, 2, \ldots, T; \sigma, \tau \geq t$, so that:

For each household $h \in \Theta$, there is a utility optimizing plan $b_n^{oh\{i(\sigma),j(\tau),t\}}$, $s_n^{oh\{i(\sigma),j(\tau),t\}}$ so that

- $b_i^{oh\{i(\sigma),j(\tau),t\}} = s_j^{oh\{i(\sigma),j(\tau),t\}} \cdot q_j^{o\{i(\sigma),j(\tau),t\}}$ (budget balance), for each $i(\sigma), j(\tau), t$

For each $i(\sigma)$, $j(\tau)$, $i(\sigma) \neq j(\tau)$,

- $\sum_{h\in\Theta} b_n^{oh\{i(\sigma),j(\tau),t\}} \leq \sum_{h\in\Theta} s_n^{oh\{i(\sigma),j(\tau),t\}}$, $n = i, j; \sigma, \tau \geq t$ (market clearing),

For $i(\sigma), i = 1, ..., N; j(\tau), j = 1, 2, ..., N; i(\sigma) \neq j(\tau); i(\sigma), j(\tau) \neq m, \sigma = t$,

- $\delta \times \sum_{h\in\Theta}[s_i^{oh\{i(\sigma),j(\tau),t\}} + s_j^{oh\{i(\sigma),j(\tau),t\}}]$
  $= \sum_{h\in\Theta}([s_i^{oh\{i(\sigma),j(\tau),t\}} - b_i^{oh\{i(\sigma),j(\tau),t\}}] + [s_j^{oh\{i(\sigma),j(\tau),t\}} - b_j^{oh\{i(\sigma),j(\tau),t\}}])$
  (transaction cost coverage)

For $i = 1, ..., N; i \neq m; \sigma = t$,

- $\delta \times \sum_{h\in\Theta}[s_i^{oh\{i(\sigma),m(t),t\}}]$
  $= \sum_{h\in\Theta}([s_i^{oh\{i(\sigma),m(t),t\}} - b_i^{oh\{i(\sigma),m(t),t\}}] + [s_m^{oh\{i(\sigma),m(t),t\}} - b_m^{oh\{i(\sigma),m(t),t\}}])$
  (transaction cost coverage).

For the case $\sigma \neq t$, similar expressions apply with ϵ substituted above for δ .

The concluding expressions are (linear) marginal cost pricing conditions; each trading post should cover its costs through the difference in goods bought (at bid price) and sold (at ask price).

The budget balance requirement applies at each transaction at each trading post. Thus, a household acquiring good $j(\tau)$ for i at $\{i(\sigma), j(\tau), t\}$ and retrading $j(\tau)$ at $\{j(\tau),\ k, t'\}$ is acquiring $j(\tau)$ at its ask price (in terms of $i(\sigma)$) at $\{i(\sigma), j(\tau), t\}$ and delivering $j(\tau)$ at its bid price (in terms of k) at $\{j(\tau), k, t'\}$. In that sequence of trades, the trader experiences — and pays — $j(\tau)$'s bid/ask spread.

8 Monetary Equilibrium

Market clearing bid prices for a monetary equilibrium appear in Section 6. In this array, good $\boldsymbol{m}$ — with the narrowest prevailing bid/ask spread — is the most liquid (saleable) good, Menger's candidate for commodity money.

The array of equilibrium trades follows. Let $\oplus$ indicate 'addition mod N' :

For $i, j = 1, 2, 3, 4, ..., N; i \neq j; j \in \{i \oplus 1, i \oplus 2, \ldots, i \oplus \Omega\}; i, j \neq m;$,

$s_i^{o[i(\sigma),j(\tau)]\{i(\sigma),m(\sigma),\sigma\}} = \rho^\sigma$,

$b_{m(\sigma)}^{o[i(\sigma),j(\tau)]\{i(\sigma),m(\sigma),\sigma\}} = (1-\delta)\rho^\sigma$,

$s_{m(\sigma)}^{o[i(\sigma),j(\tau)]\{m(\sigma),m(\tau),\sigma\}} = (1-\delta)\rho^\sigma$,

$b_{m(\tau)}^{o[i(\sigma),j(\tau)]\{m(\sigma),m(\tau),\sigma\}} = \rho^{(\tau-\sigma)}(1-\delta)\rho^\sigma = (1-\delta)\rho^\tau$,

$s_{m(\tau)}^{o[i(\sigma),j(\tau)]\{j(\tau),m(\tau),\tau\}} = \rho^{(\tau-\sigma)}(1-\delta)\rho^\sigma = (1-\delta)\rho^\tau$,

$b_j^{o[i(\sigma),j(\tau)]\{j(\tau),m(\tau),\tau\}} = \rho^{(\tau-\sigma)}(1-\delta)\rho^\sigma = (1-\delta)\rho^\tau$.

For the case $i(\sigma) = m(\sigma)$, we have

$$s_{m(\sigma)}^{o[m(\sigma),j(\tau)]\{m(\sigma),m(\tau),\sigma\}} = \rho^{\sigma}$$

$$b_{m(\tau)}^{o[m(\sigma),j(\tau)]\{m(\sigma),m(\tau),\sigma\}} = \rho^{\sigma}\rho^{(\tau-\sigma)} = \rho^{\tau}$$

$$s_{m(\tau)}^{o[m(\sigma),j(\tau)]\{m(\tau),j(\tau),\tau\}} = \rho^{\sigma}\rho^{(\tau-\sigma)} = \rho^{\tau}$$

$$b_{j}^{o[m(\sigma),j(\tau)]\{m(\tau),j(\tau),\tau\}} = (1-\delta)\rho^{\tau}$$

For the case $j(\tau) = m(\tau)$

$$s_{i}^{o[i(\sigma),m(\tau)]\{i(\sigma),m(\sigma),\sigma\}} = \rho^{\sigma}$$

$$b_{m(\sigma)}^{o[i(\sigma),m(\tau)]\{i(\sigma),m(\sigma),\sigma\}} = (1-\delta)\rho^{\sigma}$$

$$s_{m(\sigma)}^{o[i(\sigma),m(\tau)]\{m(\sigma),m(\tau),\sigma\}} = (1-\delta)\rho^{\sigma}$$

$$b_{m(\tau)}^{o[i(\sigma),m(\tau)]\{m(\sigma),m(\tau),\sigma\}} = (1-\delta)\rho^{(\tau-\sigma)}\rho^{\sigma} = (1-\delta)\rho^{\tau}$$

How does trade proceed in this setting? Household $[i(\sigma),j(\tau)]$, $(i \neq m \neq j)$ wants to trade his endowment of $i(\sigma)$ for his desired good $j(\tau)$.

In the case $\sigma = \tau$, the transactions can all take place on spot markets. Because of the low cost of monetary trade, and the high cost of barter in the absence of double coincidence of wants, the trade will take place in spot monetary terms. $[i(\sigma)$,j$(\sigma)]$ delivers good i to trading post $\{i(\sigma), m(\sigma), \sigma\}$ in exchange for $m(\sigma)$. He then goes to $\{j(\sigma), m(\sigma), \sigma\}$ delivering $m(\sigma)$ in exchange for the $j(\sigma)$ he actually wants. This is the low cost/low bid-ask spread arrangement for two related reasons. $\boldsymbol{m}$ is the low transaction cost good with correspondingly narrow bid-ask spreads. In the absence of double coincidence of wants, all other spot trade of goods i and j is also going through the trading posts for good m. That means that each side of the trade needs to carry only part of the total transaction cost, and the bid prices are structured accordingly making trade at posts $\{i(\sigma), m(\sigma), \sigma\}$ and $\{j(\sigma), m(\sigma), \sigma\}$ low cost and attractive.

In the more general case $\sigma \neq \tau$, debt markets in good $\boldsymbol{m}$ come into play. The simpler case is $\sigma < \tau$. Then household $[i(\sigma)$,$j(\tau)]$ delivers $i(\sigma)$ to the spot market $\{i(\sigma), m(\sigma), \sigma\}$ and receives m(σ) in exchange. Then the household goes to the debt market $\{m(\sigma), m(\tau), \sigma\}$ and delivers the m(σ) he has just acquired in exchange for m(τ). As noted above, the price of m(τ) is discounted reflecting the interest return on buying debt to be repaid $(\tau - \sigma)$ periods in the future. The household now owns m(τ); the household is a creditor with a claim on m deliverable at τ. $m(\tau)$ is a bond payable at τ. Some periods later, at date τ, the household enters the spot market $\{m(\tau), j(\tau), \tau\}$ and trades m(τ) for $j(\tau)$. Of the many choices household $[i(\sigma)$,$j(\tau)]$ has to implement his desired trade of $i(\sigma)$ for $j(\tau)$, he has been guided by the price system to choose this route. Direct trade of $i(\sigma)$ for $j(\tau)$ is apparently possible; it is priced at trading post $\{i(\sigma), j(\tau), \sigma\}$. But the transaction cost there is high, reflecting the high cost of futures markets and the thinness of the market (so that the transactor on one side of the market has to absorb all transaction costs). The alternative is to concentrate trade on spot markets with their lower transaction costs and on the futures market for good $\boldsymbol{m}$ with its posited low transaction cost.

The case $\sigma > \tau$ is similar, but the household becomes a debtor rather than a creditor. Household $[i(\sigma)$,$j(\tau)]$ at date τ enters the debt trading post $\{m(\tau), m(\sigma), \tau\}$ where he sells m(σ) for m(τ); he's borrowing $\boldsymbol{m}$ and promising to repay with interest at σ. He then goes to the spot trading post $\{m(\tau), j(\tau), \tau\}$ and trades m(τ) for the $j(\tau)$ he desires. The household now has a negative holding of m(σ); it is a debtor. Some periods later, at date σ, the household goes to trading post $\{i(\sigma), m(\sigma), \sigma\}$ delivering $i(\sigma)$ in exchange for spot m(σ), fulfilling its debt obligation by offsetting its earlier negative holding of $m(\sigma)$. Of the many choices household $[i(\sigma)$,$j(\tau)]$ has to implement his desired trade of $i(\sigma)$ for $j(\tau)$, he has been guided by the price system to choose this route. Direct trade of $i(\sigma)$ for $j(\tau)$ is apparently possible; it is priced at trading post $\{i(\sigma), j(\tau), \tau\}$. But the transaction cost there is high, reflecting the high cost of futures markets and the thinness of the market (priced so that a transactor on either side of the market has to absorb transaction costs both for delivering $i(\sigma)$ and acquiring $j(\tau)$). The alternative is to concentrate trade on spot markets with their lower transaction costs and on the futures market for good $\boldsymbol{m}$ with its posited low transaction cost. These markets are thick, with agents active on both sides of the trading post, so that each transactor needs to deal with transaction costs on only one side of the exchange.

In this class of examples, the markets clear, trivially, inasmuch as complete symmetry between inelastic supply and demand for each good has been assumed. The arrangement is a market clearing equilibrium with all trade going through good $\boldsymbol{m}$. Good $\boldsymbol{m}$ acts as medium of exchange, commodity money. Futures markets in $\boldsymbol{m}$ constitute borrowing and lending in the common medium of exchange.

In equilibrium, all trading posts $\{i(\sigma), j(\tau), t\}$, $i, j \neq m$, except those dealing in good $\boldsymbol{m}$ become inactive. All trading posts at date t, for $\sigma, \tau \geq t$, are priced. But active trade is transacted only at the $N-1$ posts dealing in $\boldsymbol{m}$ and the $T-t$ posts dealing in the trade of spot $\boldsymbol{m}$ for $\boldsymbol{m}$-futures. The trading posts clear. Good $\boldsymbol{m}$ has become the common medium of exchange, commodity money, and its debt instruments have become a store of value.

9 Market Clearing

The market clearing condition applies at each trading post separately. For each trading post $\{i(\sigma), j(\tau), t\}$ the sum of buying less selling orders for each good traded there should be nonpositive. That is,

$$\sum_{h \in \Theta} [b_i^{h\{i(\sigma),j(\tau),t\}} - s_i^{h\{i(\sigma),j(\tau),t\}}] \leq 0 \ .$$

In the monetary equilibrium described above, most trading posts are inactive, so the market clearing condition at inactive posts is trivially fulfilled. The most active trading posts are the money market posts, $\{m(\sigma), m(\tau), \sigma\}$. The specification of Θ is sufficiently symmetric in its timing of σ and τ that market clearing follows. For every borrower, there is a lender; for every disbursement of spot money there is an acquisition of spot money. The money market clears.

9.1 Clearing spot m at trading post $\{m(t), i(t), t\}$:

For each $h \in \Theta, i \neq m$, the budget constraint says

$b_i^{oh\{i(t),m(t),t\}} = s_m^{oh\{i(t),m(t),t\}} \cdot q_m^{o\{i(t),m(t),t\}}$ where $q_m^{o\{i(t),m(t),t\}} = 1$, and

$b_m^{oh\{i(t),m(t),t\}} = s_i^{oh\{i(t),m(t),t\}} \cdot q_i^{o\{i(t),m(t),t\}}$ where $q_i^{o\{i(t),m(t),t\}} < 1$.

Market clearing in good i, allowing for transaction costs of $(1-q_i^{o\{i(t),m(t),t\}})$ per unit i traded, gives $\sum_{h\in\Theta}[b_i^{oh\{i(t),m(t),t\}} - q_i^{o\{i(t),m(t),t\}} \cdot s_i^{oh\{i(t),m(t),t\}}] = 0$

The Walras' Law at trading post $\{i(t), m(t), t\}$ is

$\sum_{h\in\Theta}[b_i^{oh\{i(t),m(t),t\}} - s_m^{oh\{i(t),m(t),t\}} + b_m^{oh\{i(t),m(t),t\}} - q_i^{o\{i(t),m(t),t\}} \cdot s_i^{oh\{i(t),m(t),t\}}] = 0$.

Thus

$\sum_{h\in\Theta}[b_m^{oh\{i(t),m(t),t\}} - s_m^{oh\{i(t),m(t),t\}}] = 0$, market clearing in the spot money market with good i.

9.2 Clearing the money market trading posts

We have

for each τ, $\sigma = 1, 2, \ldots, T, \sigma \leq \tau$

$$\sum_{h=[i(\sigma),j(\tau)]} b_{m(\tau)}^{oh\{m(\sigma),m(\tau),\sigma\}} = \sum_{h=[i(\sigma),j(\tau)]} \rho^{\sigma}(1-\delta)$$

$$\sum_{h=[i(\tau),j(\sigma)]} s_{m(\tau)}^{oh\{m(\sigma),m(\tau),\sigma\}} = \sum_{h=[i(\tau),j(\sigma)]} \rho^{\tau}(1-\delta)$$

Summing it up for trading post $\{m(\sigma), m(\tau), \sigma\}$

$$\sum_{h\in\Theta} b_{m(\tau)}^{oh\{m(\sigma),m(\tau),\sigma\}} - \sum_{h\in\Theta} s_{m(\tau)}^{oh\{m(\sigma),m(\tau),\sigma\}} = 0$$

The goods markets clear similarly, reflecting the symmetry of timing of demands and supplies in the specification of Θ.

10 Debt markets and the Rate of Interest

The futures contract $m(\tau)$ at date $t < \tau$ is a debt instrument. Some households will hold positive quantities, others negative quantities. In a market clearing equilibrium, the holdings will sum to zero. $q_{m(t)}^{\{m(t),m(\tau),t\}} = \rho^{(t-\tau)}$ is the value of a unit $m(\tau)$ note at date t payable at τ. The implicit rate of interest is $(\rho - 1)$ per period.

11 The quantity of money

The constraint (T.iii) requires that terminal asset holdings be nonnegative. But it does not constrain asset positions in earlier periods. Thus households may have positive or negative notes outstanding throughout periods 1 through T, resolving them eventually by the terminal horizon. In a model including banks and financial institutions, these holdings might well be measured as part of the money supply.

Moreover, there is no cash-in-advance constraint in this model. This is a pure flow model, taking little account of available stocks held. One does not need to own commodity $\boldsymbol{m}$ one period prior to trading it. It is sufficient to acquire, trade, and repay outstanding debts denominated in $\boldsymbol{m}$ all in a single period, without ever holding it as a stock. Thus, the quantity of $\boldsymbol{m}$, though well defined, does not reflect its availability or usefulness in trade. More indicative is the volume of commodity $\boldsymbol{m}$ transactions, $\sum_{h\in\Theta} b_m^{oh\{i(\sigma),m(\sigma),\sigma\}}$ at any single date σ.

11.1 What's wrong with this picture?

Clearly, there's a great deal left out of this class of examples. There is no discussion here of how $\boldsymbol{m}$, 'money', generates a peculiarly low transaction cost. The notion, mentioned above, that one can trade $\boldsymbol{m}$, 'money', without ever holding on to it for any finite time is clearly an oversimplification. The velocity of circulation is undefined or may be arbitrarily large. Though not empirically correct, this velocity does approximate the character of some financial transactions, where balances are netted out at the end of the day and daylight overdrafts regularly occur.

11.2 What's right with this picture?

Developing a model — with the formal generality of the Arrow-Debreu general equilibrium model, Debreu (1959) — of monetary trade over time and at each point in time is a daunting task, Hahn (1971), Starrett (1973). The structure of prices and price expectations presented here is typical of a sequence economy, Radner (1972), Hahn (1971).

At each date there is a full set of spot and futures markets and the future array of spot and futures prices is correctly foreseen. Thus price expectations are fully rational.

12 Conclusion

Tobin (1961, 1980) and Hahn (1982) despaired of achieving a general equilibrium model based on elementary price theory resulting in a common medium of exchange. But the price array in Section 6 leads directly to a monetary equilibrium in Section 8. Monetary trade is the result of decentralized optimizing decisions of households guided by prices without government, central direction, or fiat. The price system provides all the co-ordination required to maintain a common medium of exchange and a debt (and asset) market in that medium (hence a store of value). Of course, we expect successful decentralized co-ordination in an Arrow-Debreu Walrasian general equilibrium model, Debreu (1959). But the Arrow-Debreu model is framed for a non-monetary economy. The example here demonstrates — as Menger (1892) argued — that the price system can generate a monetary equilibrium with a single common intertemporal carrier of value and medium of exchange.

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