Consider a small economy, with two goods and three households. The two goods are denoted x, y. The households have identical preferences on $\mathbf{R}^2_+$ described by the utility function

$$u(x, y) = \sup[x, y]$$

where "sup" indicates the supremum or maximum of the two arguments. These tastes could be characterized by the household saying

"I like x and y equally well, and more is definitely better. But they are redundant. When there's more x, I use the x and discard the y. And when there's more y, it's y that I use and discard the x."

The households have identical endowments of (10, 10).

(i) Demonstrate that there is no competitive equilibrium in this economy [Hint: Show that price vector $(\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon)$, $\varepsilon > 0$, cannot be an equilibrium; similarly for $(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon)$; and finally $(\frac{1}{2}, \frac{1}{2})$. That pretty well takes care of it.]

(ii) The standard results for an Arrow-Debreu general equilibrium model include proofs of existence of general equilibrium. That result apparently fails in the example above. Explain. How can this happen? Is the example above a counterexample, demonstrating that the usual existence of general equilibrium results are invalid? Does the example above fulfill the usual sufficient conditions for existence of general equilibrium in an Arrow-Debreu model?