Midterm Exam 1 : Suggested Answers

A1, B4, C4, D1

Consider a two-person pure exchange economy (an Edgeworth Box). Let there be two households, 1 and 2 with different endowments. Superscripts are used to denote the name of the households. There are two commodities, x and y.

Household 1's utility function is denoted as $u^1(x^1, y^1)$, and 1 has endowment $r^1 = (8, 0)$. Household 2's utility function is $u^2(x^2, y^2)$, with endowment $r^2 = (2, 10)$. (The superscripts are household names, not powers)
A competitive equilibrium consists of prices $p^\circ = (p^\circ_x, p^\circ_y)$ and allocation $(x^{\circ 1}, y^{\circ 1})$, $(x^{\circ 2}, y^{\circ 2})$ so that

(a) Household 1's consumption plan $(x^{\circ 1}, y^{\circ 1})$ maximizes $u^1(x, y)$ subject to household 1's budget constraint, $p^\circ_x x + p^\circ_y y = 8p^\circ_x$; and similarly

(b) Household 2's consumption plan $(x^{\circ 2}, y^{\circ 2})$ maximizes 2's utility subject to 2's budget, $p^\circ_x x + p^\circ_y y = 2p^\circ_x + 10p^\circ_y$; and

(c) $(x^{\circ 1}, y^{\circ 1}) + (x^{\circ 2}, y^{\circ 2}) = (8, 0) + (2, 10) = (10, 10)$

(d) In addition, assuming that $x^{\circ 1}, y^{\circ 1}, x^{\circ 2}, y^{\circ 2} > 0$, 1's and 2's respective MRS's at competitive equilibrium consumption levels can be characterized as

$$MRS^1_{xy} = \frac{\frac{\partial u^1}{\partial x}}{\frac{\partial u^1}{\partial y}} = \frac{p^\circ_x}{p^\circ_y} = \frac{\frac{\partial u^2}{\partial x}}{\frac{\partial u^2}{\partial y}} = MRS^2_{xy}$$

In a competitive equilibrium: (i) markets are supposed to clear, equating supply and demand;
(ii) the resulting allocation of goods is supposed to achieve a Pareto efficient allocation to the households; (iii) each household optimizes it's utility subject to budget constraint.

1. Which of characteristics (a), (b), (c), (d), represents property (i)? Explain briefly.

2. Which of characteristics (a), (b), (c), (d), represents property (ii)? Explain briefly.

3. Which of characteristics (a), (b), (c), (d), represents property (iii)? Explain briefly.

Suggested Answer:

1. (c) represents the idea of market clearing (i), showing that demands (the left hand side) equal supplies (the right hand side).

2. (d) represents Pareto efficiency (ii), equating marginal rates of substitution of the two goods across households.

3. (a) and (b) present (iii) utility maximization subject to budget constraint.

A2, B1, C3, D4

The Brouwer Fixed Point Theorem can be stated as:

Let $K \subset \mathbf{R}^{\mathbf{N}}$. K nonempty, compact, and convex. Let $f(\cdot)$ be a continuous function, $f : K \to K$. Then there is $x^* \in K$ so that $f(x^*) = x^*$.

1. Let $K = (0, 1)$ the open unit interval in $\mathbf{R}$, and $f : K \to K$, f continuous on K. Does the Brouwer Fixed Point Theorem apply in this setting? Explain
2. Let $K = (0, 1)$ the open unit interval in $\mathbf{R}$, and $f : K \to K$, f continuous on K. Is there a unique fixed point of f in K? Is every point of K a fixed point of f? Is there no fixed point of f in K? Explain.

Suggested Answer:

1. No. The open interval is not closed and hence not compact. The Brouwer FPT simply does not apply.
2. All of the suggestions are possibilities. There may be zero ($f(x) = .5x$), one ($f(x) = .5$), or infinite ($f(x) = x$)fixed point(s). The Brouwer Fixed Point Theorem gives no guidance.

A3, B2, C1, D2

A statment and proof of Theorem 5.2 (Existence of Competitive Equilibrium in an Economy with a Continuous Excess Demand Function fulfilling Walras's Law) is in the appendix. This is the same treatment presented in the textbook and in class. At the point of the proof where it is established that $T(p^*) = p^*$ there is a line inserted saying "Why not stop the proof here?" Explain why the proof needs to continue. [Hint: If you think the answer is too obvious to be right, don't worry. It is obvious.]

Suggested Answer: The way the mapping T is constructed, any general equilibrium price vector will be a fixed point of T, but it does not follow directly that that (those) is (are) the only fixed point(s). The next, and necessary, step is to prove that the fixed point p^* is a market-clearing equilibrium price.

A sufficient answer is to quote the proof: "Now we have to show that the auctioneer's decision to stop adjusting the price is really the right thing to do. That is, we'd like to show that p^* is not just the stopping point of the price adjustment process, but that it actually does represent general equilibrium prices for the economy."

A4, B3, C2, D3

For each of the following sets, state and explain briefly whether it is open, closed, bounded, compact, convex:

1. $S = \{x \in \mathbf{R^N} | \mathbf{5 \leq |x| \leq 10}\}$
2. $\mathbf{R^N}$
3. $T = \{0\}$ where 0 is the origin in $\mathbf{R^N}$

Suggested Answer:

1. S is not open (does not contain a neighborhood of included boundary points like $(10, 0, ..., 0)$); is closed (includes its boundary points); is bounded (can be contained in cube of side 20 centered at the origin); is compact (closed and bounded); is not convex (there's a big hole in the middle).
2. $\mathbf{R^N}$ is open (contains a neighborhood of every point in the set); is closed (contains its limit points); unbounded ($\mathbf{R^N}$ is infinite, cannot be contained in a finite cube); not compact because unbounded; is convex (there are no holes or indentations; the line segment between any two points of $\mathbf{R^N}$ is contained in $\mathbf{R^N}$).
3. T is not open (does not contain a neighborhood of $0 \in T$); is closed (contains its only limit point, 0); is bounded (is contained in any cube of positive side centered at the origin); compact since it is closed and bounded; is convex since it is only a single point (the line segment joining any two points 0 and 0 of T is a subset of T).