Lecture Notes of February 27, 29: Production with unbounded technology

General Equilibrium in an Economy with unbounded technology sets

Delete P.VI (bounded $\mathcal{Y}^j$). Like all good mathematicians, we're reducing this to the previous case. Replace $\mathcal{Y}^j$ with Y^j. Define $Y \equiv \sum_{j \in F} Y^j$. Let firm j's (unbounded) production technology be Y^j. Define $S^j(p)$ (no tilde) as j's profit maximizing supply in Y^j. Define $D^i(p)$ as i's demand without restriction to $\{x||x| \leq c\}$.

Under assumptions of No Free Lunch (P.IV(a)) and Irreversibility (P.IV(b)), the attainable output set for the economy and for each firm is still bounded.

P.IV. (a) if $y \in Y$ and $y \neq 0$ then $y_k < 0$ for some k. (b) if if $y \in Y$ and $y \neq 0$ then $-y \notin Y$.

Define $\tilde{Y}^j = Y^j \cap \{x||x| \leq c\} \equiv \mathcal{Y}^j$ in chapters 11 - 14. Define $\tilde{S}^j(p)$ as j's supply function based on $\tilde{Y}^j$.

We restate for the technologies Y^j the assumptions P.I–P.III on production technologies introduced in Chapter 11 for the technology sets $\mathcal{Y}^j$:

(P.I) Y^j is convex for each $j \in F$.
(P.II) $0 \in Y^j$ for each $j \in F$.
(P.III) Y^j is closed for each $j \in F$.

The aggregate technology set is $Y = \sum_{j \in F} Y^j$.

(P.IV)(a) if $y \in Y$ and $y \neq 0$, then $y_k < 0$ for some k (No Free Lunch).

(b) if $y \in Y$ and $y \neq 0$, then $-y \notin Y$ (irreversibility).

P.IV is not an assumption about the individual firms; it treats the production sector of the whole economy.

$r \in \mathbf{R}_+^N$ = vector of total initial resources or endowments.

Definition Let $y \in Y$. Then y is said to be attainable if $y + r \geq 0$ (the inequality holds co-ordinatewise).

In an attainable production plan $y \in Y$, $y = y^1 + y^2 + \ldots + y^{\#F}$, we have $y + r \geq 0$. But an individual firm's part of this plan, y^j , need not satisfy $y^j + r \geq 0$. Thus

Definition We say that $y^j \in Y^j$ is attainable in Y^j if there exists a $y^k \in Y^k$ for each of the firms $k \in F$, $k \neq j$, such that $y^j + \sum_{k \in F, k \neq j} y^k$ is attainable.

Lemma 15.1 Assume P.II and P.IV. Let $y = \sum_{j \in F} y^j,\ y^j \in Y^j$ for all

$j \in F,\ y \in Y,\ y = \mathbf{0}$. Then $y^j = \mathbf{0}$ for all $j \in F$.

Theorem 15.1 For each $j \in F$, under P.I, P.II, P.III, and P.IV, the set of vectors attainable in Y^j is bounded.

Theorem 15.2 Under P.I–P.IV, the set of attainable vectors in Y is compact, that is, closed and bounded.

Theorem 15.3(b): If $S^j(p)$ is attainable in Y^j, then $S^j(p) = \tilde{S}^j(p)$.

Theorem 16.1(b): If $M^i(p) = \tilde{M}^i(p)$, and $D^i(p)$ is attainable, then $D^i(p) = \tilde{D}^i(p)$.

$Z(p) = \sum_{i \in H} D^i(p) - \sum_{j \in F} S^j(p) - \sum_{i \in H} r^i$

Theorem 18.1: Assume P.II-P.V, and C.I-C.V, C.VI(SC), CVII. There is $p* \in P$ so that $p*$ is an equilibrium price vector. That is, $Z(p*) \leq 0$ (the inequality holds co-ordinatewise) with $p*_k = 0$ for k so that $Z(p*) < 0$.

Proof: The artificially bounded economy characterized by production technologies $\tilde{Y}^j, j \in F$ fulfills all of the assumptions of the bounded technology economy of chapters 11 - 14. Find an equilibrium of that bounded economy. Its equilibrium allocation is attainable so by Theorems 15.3(b) and 16.1(b) the bounded and unbounded supply functions coincide and the bounded and unbounded demand functions coincide. Equilibrium prices of the bounded economy exist and are equilibrium prices of the unbounded economy with technology sets Y^j. QED

Theorem 18.1 here is the most important single result of this course. It says that the competitive economy, guided only by prices, has a market clearing equilibrium outcome. The decentralized price-guided economy has a consistent solution. This is the defining result of the general equilibrium theory.