Suggested Answers

Please answer all questions. Each of the four questions marked with a big number counts equally. Designate your answers clearly. Notation not otherwise specified is taken from *General Equilibrium Theory: An Introduction 2nd edition.*

1 Supply Function, $\tilde{S}^j(p)$

Please refer to the lecture notes of February 1 & 3, including the statement and proof of Theorem 11.1. Consider the three descriptions below explaining how assumptions P.III and P.VI enter the proof. Which one (or several or none) most correctly describes their function. Explain below.

Circle the response(s) that is(are) valid. Then explain your answer below:

(i) P.III and P.VI provide a convenient simplification, keeping $\tilde{S}^j(p)$ within a ball of radius c. They are not directly used in the proof of Theorem 11.1.

(ii) P.III helps to establish that $\tilde{S}^j(p)$ is point-valued. Without P.III, $\tilde{S}^j(p)$ might be a convex subset (a flat segment on the boundary) of $\mathcal{Y}^j$ with many equally profitable elements at prices p.

(iii) P.III and P.VI (along with P.II) together imply non-emptiness of $\tilde{S}^j(p)$. The proof of continuity of $\tilde{S}^j(p)$ as a function of p also uses P.III and P.VI.

Explain your answer:

Suggested Answer: (iii) is correct. P.III and P.VI ensure that $\mathcal{Y}^j$ is a compact set. That, combined with P.II, assures that the maximization defining $\tilde{S}^j(p)$ has a result, assuring non-emptiness of $\tilde{S}^j(p)$. Further, the compactness of $\mathcal{Y}^j$ is used to prove continuity of $\tilde{S}^j(p)$ in the proof of Theorem 11.1, by assuring that the sequence $\tilde{S}^j(p^\nu)$ has a convergent subsequence.

2 Competitive Equilibrium

Consider the following (completely conventional) definition: $\{p^\circ, x^{\circ i}, y^{\circ j}\}$, $p^\circ \in \mathbf{R}^N_+$, $i \in H$, $j \in F$, *is said to be a competitive equilibrium if*

(i) $y^{\circ j} \in Y^j$ *and* $p^\circ \cdot y^{\circ j} \geq p^\circ \cdot y$ *for all* $y \in Y^j$, *for all* $j \in F$,

(ii) $x^{\circ i} \in X^i$, $p^\circ \cdot x^{\circ i} \leq M^i(p^\circ) = p^\circ \cdot r^i + \sum_{j \in F} \alpha^{ij} p^\circ \cdot y^{\circ j}$ *and* $u^i(x^{\circ i}) \geq u^i(x)$ *for all* $x \in X^i$ *with* $p^\circ \cdot x \leq M^i(p^\circ)$ *for all* $i \in H$, *and*

(iii) $0 \geq \sum_{i \in H} x^{\circ i} - \sum_{j \in F} y^{\circ j} - \sum_{i \in H} r^i$ *with* $p^\circ_k = 0$ *for coordinates* k *so that the strict inequality holds.*

1. The concept of competitive equilibrium is supposed to reflect price-taking behavior. Households and firms are supposed to treat prices 'parametrically,' as values they adjust to but cannot choose.

   Circle here the number of which part(s) — (i), (ii), or (iii) — (perhaps more than one) represents the concept of price-taking. Explain your choice here:

   Suggested Answer: (i) and (ii) represent price-taking behavior, as firms (i) and households (ii) optimize subject to fixed, given, prices treated as information that they use but cannot affect.

2. The concept of competitive equilibrium is supposed to reflect market-clearing. Supplies and demands for all commodities are equated, with the possible exception of free goods in excess supply in equilibrium.

   Circle here the number of which part(s) — (i), (ii), or (iii) — (perhaps more than one) represents the concept of market clearing. Explain your choice here:

   Suggested Answer: (iii) is correct. The inequality says that no goods are in excess demand and that those in excess supply have a price of zero.

3 Edgeworth Box

A two-person, two-commodity, pure exchange (no production) economy is known as an Edgeworth box. We would like to use Theorem 14.1 (stated and proved in lecture notes of February 15, 17, & 22) to demonstrate the existence of competitive equilibrium in an Edgeworth box. Set $\mathcal{Y}^j \equiv \{0\}$ for all $j \in F$, where 0 is the zero vector in $\mathbf{R}^N$.

(i) The production sector of this economy is represented as $\mathcal{Y}^j \equiv \{0\}$ for all $j \in F$. How can this specification represent the case of a pure exchange (no production) economy (This is not a trick question — if you think the answer is obvious, that's because it is.)? Explain. **Enter your answer here:**

Suggested Answer:$\mathcal{Y}^j \equiv \{0\}$ for all $j \in F$ means that for the production sector there are no inputs and no outputs and precisely zero profits. So production enters no one's consumption nor income. That's a pure exchange economy, an economy with no production.

(ii) As a single point, $\mathcal{Y}^j \equiv \{0\}$ is trivially strictly convex (P.V).
Is $\mathcal{Y}^j \equiv \{0\}$ closed (P.III)? **Yes / No** (circle one). **Explain:**

Suggested Answer: Yes. A set consisting of a single point necessarily contains all of its limit points.

Is $\mathcal{Y}^j \equiv \{0\}$ bounded (P.VI)? **Yes / No** (circle one). **Explain:**

Suggested Answer: Yes. A set consisting of the origin only is necessarily bounded. It can be contained in a cube centered at the origin.

Are the usual assumptions on production fulfilled by this choice of $\mathcal{Y}^j$? **Yes / No** (circle one). **Explain:**

Suggested Answer: Yes. The usual assumptions are P.II, contains the origin; P.III, closed; P. V, strictly convex; P.VI, bounded. They are all fulfilled by $\mathcal{Y}^j \equiv \{0\}$ for all $j \in F$.

(iii) Can Theorem 14.1 then be applied to give sufficient conditions for existence of market clearing prices in the Edgeworth Box? **Yes / No** (circle one). **Explain:**

Suggested Answer: Yes. As stated in part (ii) all of the assumptions on the production side of the economy of Theorem 14.1 are fulfilled by this choice of production technology. Part (i) shows that this is a suitable representation of a pure exchange economy. Hence Theorem 14.1 can be applied. It's assumptions give sufficient conditions on the household side to assure existence of general equilibrium prices.

4 U-shaped cost curves

The usual U-shaped cost curve model of undergraduate economics includes a small nonconvexity (diminishing marginal cost at low output levels). This is a violation of our usual convexity assumptions on production (P.I or P.V). Consider the general equilibrium of an economy displaying U-shaped cost curves. Which (one or more or none) of the following statements is valid? Explain.

Circle the response(s) that is(are) valid. Then explain your answer below:

(i) There is surely a competitive equilibrium. The model is used in Econ 1 and Econ 100 as a demonstration of competitive equilibrium behavior.

(ii) There may be a competitive equilibrium, but since the sufficient conditions for existence of equilibrium are violated, it is difficult to be sure.

(iii) There is no competitive equilibrium. The nonconvexity prevents existence of competitive equilibrium.

Explain your answer:

Suggested Answer: (ii) is correct. The nonconvexity in technology can produce jumps — discontinuities — in supply functions that may prevent the existence of general equilibrium. But since the conditions of Theorem 14.1 are sufficient conditions, not necessary conditions, it is hard to tell. There may still be general competitive equilibrium prices. In the absence of sufficient conditions we can't be sure.