

Econ 113, Midterm 2, Take-home section

This examination is take-home, open-book, open-notes. There is no time limit other than the due date. You may consult any published source (cite your references). Other people are closed. Do not discuss with classmates, friends, professors (except with Troy or Prof. Starr — who promise to be clueless), until the examination is collected. If you have questions, e-mail them to Prof. Starr at rstarr@ucsd.edu.

Notation not defined here is taken from Starr's *General Equilibrium Theory*. State any additional assumptions you need. Each of the questions has a long discussion followed by the real question (the only part you should answer) denoted by a bullet •.

1

Recall

Intermediate Value Theorem Let $[a, b]$ be a closed interval in \mathbf{R} and f a continuous real-valued function on $[a, b]$ so that $f(a) < f(b)$. Then for any real c so that $f(a) < c < f(b)$ there is $x \in [a, b]$ so that $f(x) = c$.

This problem is restated from Starr's *General Equilibrium Theory*, problem 7.1. Consider a two-commodity economy with an excess demand function $\tilde{Z}(p) \equiv (\tilde{Z}_1(p), \tilde{Z}_2(p))$. The price space is $p \in P = \{p \mid p \in \mathbf{R}^2, p \geq 0, p_1 + p_2 = 1\}$. Let $\tilde{Z}(p)$ be continuous, bounded, and fulfill Walras' Law as an equality, that is $p \cdot \tilde{Z}(p) = p_1 \tilde{Z}_1(p) + p_2 \tilde{Z}_2(p) = 0$. Assume $\tilde{Z}_1(0, 1) > 0$ at $p = (0, 1)$, and $\tilde{Z}_2(1, 0) > 0$ at $p = (1, 0)$. Note that this implies for $\epsilon > 0$ sufficiently small, that $\tilde{Z}_2(\epsilon, 1 - \epsilon) < 0$, $\tilde{Z}_1(1 - \epsilon, \epsilon) < 0$.

- Use the intermediate value theorem and Walras' Law to show that the economy has a competitive equilibrium. That is, demonstrate that there is a price vector $p^* \in P$ so that $\tilde{Z}(p^*) = (0, 0)$. Hint: Characterize $\tilde{Z}(p)$ as $\tilde{Z}(\alpha, 1 - \alpha)$ for $0 \leq \alpha \leq 1$. Use the intermediate value theorem to find $0 < \alpha < 1$ so that $\tilde{Z}_1(\alpha, 1 - \alpha) = 0$. Then apply Walras' Law.

2

Related to Starr's *General Equilibrium Theory*, problem 7.9. Consider a two-person, two-commodity pure exchange economy, an Edgeworth Box, with $X^i \equiv R_+^2$ for both households. Households are named $i = 1, 2$. Assume axioms C.I - C.V, C.VII, C.VIII. This is an economy without active production so assume $\mathcal{Y}^j \equiv \{0\}$ (the set whose only element is the zero vector) for all $j \in F$. Note that this specification of \mathcal{Y}^j trivially fulfills P.II, P.III, P.V, P.VI.

Recall Theorem 7.1: Assume P.II, P.III, P.V, P.VI, C.I–C.V, C.VII, and C.VIII. There is $p^* \in P$ so that p^* is an equilibrium.

- Demonstrate that the Edgeworth Box has a competitive equilibrium price vector, p^* . Hint: Check that Theorem 7.1 applies. If so, then check that p^* clears the Edgeworth Box. That is, show that

$\tilde{D}^1(p^*) + \tilde{D}^2(p^*) = 0$ (the zero vector) or $\tilde{D}^1(p^*) + \tilde{D}^2(p^*) \leq 0$ (the zero vector; the inequality applies co-ordinatewise) with $p_k^* = 0$ for a good $k = 1, 2$, where the strict inequality holds.

3

This problem uses the tax/redistribution framework in Starr's *General Equilibrium Theory*, problem 7.2. Refer to your textbook for the setting. Note that the redistributive taxation is equivalent to considering an alternative economy with endowments differing by a transfer prior to starting economic activity. After taxation, the economy looks like the original economy with endowments revised to $\hat{r}^i \equiv 0.5r^i + (1/\#H) \sum_{h \in H} .5r^h$. You showed in Problem 7.2 that there exists a competitive equilibrium in the revised economy; assume that result here. Denote the equilibrium prices p^* . The First Fundamental Theorem of Welfare Economics (Theorem 12.1) is proved in a setting without taxation.

- Does the First Fundamental Theorem of Welfare Economics (Theorem 12.1) apply to the equilibrium allocation of the economy with taxation/redistribution, at prices p^* ? Is the general equilibrium allocation Pareto efficient? Explain.