Take-Home Midterm ODD-ODD-ODD

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Prof. Starr or Ms. Fried — who promise to be clueless), until the examination is collected.

1

Consider the general competitive equilibrium of a production economy with corporate income taxation. In addition to the prices of goods $p \in P$, there is a (scalar) corporate tax rate τ, $1 > \tau > 0$. Proceeds of the tax are then distributed to households as a lump sum. Household income then is

$M^i(p) = p \cdot r^i + \left[\sum_{j \in F} \alpha^{ij} \, (1 - \tau) \, p \cdot S^j(p)\right] + T,$

where T is the transfer of tax revenues to the household. The transfer to the typical household is

$T = \frac{1}{\#H} \sum_{j \in F} \tau(p \cdot S^j(p)).$

The household budget constraint is p$\cdot D^i(p) \leq M^i(p)$.Assume the household consumption sets are the nonnegative quadrant, R^N_+ and that household endowments are $r^i \gg 0$, (endowments are strictly positive in all goods).

1. The first part of the (Weak) Walras' Law can be stated as

   p$\cdot Z(p) = p \cdot \sum_{i \in H} D^i(p) - p \cdot \sum_{j \in F} S^j(p) - p \cdot \sum_{i \in H} r^i \leq 0.$

   Show that this part of the (Weak) Walras' Law is fulfilled. You may assume that the rest of the (Weak) Walras' Law is fulfilled as well.

2. Theorem 18.1 is proved in a model without taxation. Does there exist a competitive equilibrium in the economy with corporate income taxation? You may assume P.I - P.VI, C.I-C.VI(SC), C.VII. Explain.

2

Consider the following Edgeworth Box example.

Do all parts.

Superscripts are used to denote the name of the households *and unfortunately, raising the consumption to a squared value — we'll try to keep them straight.* Households are characterized by a utility function and an endowment vector. The possible consumption set is the nonnegative quadrant, R^2_+. There are two commodities, x and y.

Household A is characterized as $\mathrm{u}^A(x, y) = [x]^2 + [y]^2$(where the terms in brackets are raised to the power 2), with endowment $r^A = (5, 5)$. Household A's optimizing consumption subject to budget constraint will typically be a corner solution, so marginal equivalences will not be fulfilled as an equality.

Household B has the same preferences and endowment. $\mathrm{u}^B(x, y) = [x]^2 + [y]^2$ $\mathrm{r}^B = (5, 5)$.Denote A's demand as (x^A, y^A), B's as (x^B, y^B).

1. The utility functions in this Edgeworth Box example violate some of the usual assumptions C.I - C.V, C.VI(SC), C.VII. Which one(s) do they violate? Explain.

2. We claim there is however a competitive equilibrium in this Edgeworth Box. Find competitive equilibrium prices (p_x^*, p_y^*) and an equilibrium allocation.

3. Considering that the conditions of Theorem 14.1 have not been fulfilled, how is it possible that there is a competitive equilibrium? Is this a counterexample to Theorem 14.1 (that is, does it demonstrate that Theorem 14.1 is false?) ?

3

Consider an Edgeworth Box for two households. The two goods are denoted x, y. The expression $\succ$ denotes strict preference; the expression $\sim$ denotes indifference, equivalence in preference. The households have identical preferences:

$$\begin{aligned}(x, y) &\succ (x', y') \text{ if } 3x + y > 3x' + y', \text{ or}\\ (x, y) &\succ (x', y') \text{ if } 3x + y = 3x' + y' \text{ and } x > x'.\\ (x, y) &\sim (x', y') \text{ only if } (x, y) = (x', y').\end{aligned}$$

They have identical endowments of (20, 20). Demonstrate that there is no competitive equilibrium. Is this example a counterexample to Theorem 18.1 (does it demonstrate that Theorem 18.1 is false?) ? If so explain why Theorem 18.1 is false. If not, state which of Theorem 18.1's assumptions is not fulfilled and demonstrate that it is violated. (Hint: Consider demand in the neighborhood of price vector $(p_x, p_y) = (.75, .25)$.)