

# Take-Home Final Exam EVEN-ODD-ODD

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Prof. Starr or Ms. Fried — who promise to be clueless), until the examination is collected.

Please use a blue book. There are four questions. Answer all questions fully. State clearly any additional assumptions you need.

## 1

The proof of the First Fundamental Theorem of Welfare Economics (Theorem 19.1) uses either

- (i) the combined assumption of non-satiation (C.IV) and convexity (C.VI (C)) or
- (ii) the assumption of weak monotonicity of preferences (C.IV\*).

These assumptions are designed to assure local non-satiation of preferences. That is, nearby to every point in  $x \in X^i$  there is  $x' \in X^i$  so that  $u^i(x') > u^i(x)$ . Thus there are no thick indifference curves (zones of satiation). This ensures that for each household  $i \in H$ , the household spends fully up to the limits of its income

$$p^\circ \cdot w^{\circ i} = M^i(p^\circ) = p^\circ \cdot r^i + \sum_j \alpha^{ij} (p^\circ \cdot y^{\circ j}),$$

You may find looking at problems 19.4 and 19.15 useful.

Theorem 19.1 is invalid (that is, the conclusion may not be true) without one of (i) or (ii), because for some households without (i) or (ii) we may have

$$p^\circ \cdot w^{\circ i} < M^i(p^\circ) = p^\circ \cdot r^i + \sum_j \alpha^{ij} (p^\circ \cdot y^{\circ j}),$$

Explain how the proof of Theorem 19.1 would fail without C.IV and C.VI(C) or C.IV\*. Without those assumptions how could  $p^\circ \cdot w^{\circ i} < M^i(p^\circ)$  for some households? Why does the proof fail if  $p^\circ \cdot w^{\circ i} < M^i(p^\circ)$  for some households? [Hint: It is not sufficient to give an example where the equilibrium allocation is not Pareto efficient. This question asks you to look at the proof to see how C.IV and C.VI(C) (or C.IV\*) are used, and to identify which essential step(s) cannot be taken in their absence.]

**2**

This question deals with the Arrow Possibility Theorem, using the notation of the Lecture Notes of May 23. Let  $\hat{P} \in \Pi$ ; that is  $\hat{P}$  is a transitive ordering on  $X$  the space of choice options. Let  $f : \Pi^{\#H} \rightarrow \Pi$ .  $f = \text{constant} = \hat{P}$ . That is, for every preference profile,  $f$  gives the same value of  $P$ ,  $f = \hat{P}$ . The social ordering is transitive but completely independent of individual preferences. This specification of  $f$  violates one of the four Arrow conditions. Which one? Explain fully.

**3**

Starr's *General Equilibrium Theory: An Introduction* draft second edition, Problem 19.14.

**4**

Starr's *General Equilibrium Theory: An Introduction* draft second edition, Problem 18.5. Feel free to assume the results of Problem 18.4.