The Relationship between the Income Elasticities of Demand and Willingness to Pay*

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The relationship between income and willingness to pay for collectively provided public/environmental goods is investigated. We show that while the income elasticity of willingness to pay and the ordinary income elasticity of demand are related, knowledge of one is insufficient to determine the magnitude or even the sign of the other. The income elasticity of willingness to pay is influenced by additional factors which are generally unobservable. Examples are provided to illustrate the degree to which the two income elasticities may diverge. Our results indicate that even when goods are demand luxuries they may or may not have income elasticities of willingness to pay which are greater than one.   © 1997 Academic Press

I. INTRODUCTION

When considering environmental policies, economists typically focus on economic efficiency even though there are usually important distributional implications as well. Depending upon the distribution of costs and benefits, policies may be regressive in the sense that net benefits are larger for individuals with high incomes than for those with lower incomes. The degree of regressiveness may in turn limit a policy's political appeal. In contrast, the case for government intervention is strengthened in situations where net benefits are positive and large across income groups. Distributional issues are not limited to a specific country or point in time. In debates over reforms that will affect other countries or generations, identifying potential winners and losers, and in particular how net benefits will be distributed by income level, figures prominently.

Among economists, there has been considerable discussion over whether environmental protection disproportionately favors the wealthy at the expense of lower income groups or countries even though there is little evidence that this is in fact the case.[1] The "inverted-U" curve findings linking income to environmental

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[1] For discussions see Harry *et al*. [7], Freeman [4], Tucker [23], Pearce [17], and Baumol and Oates [1].

quality[2] offer evidence that the net benefits of environmental regulation may only become positive at higher income levels. However, these authors make clear that the observed trajectories for environmental quality reflect the outcomes of social choice processes which may be difficult to link to individual preferences.

Evidence that is more easily linked to individual preferences and behavior can be found in travel cost studies which typically find income elasticities for changes in recreational consumer surplus less than one [15]. Perhaps the strongest empirical evidence linking income and environmental quality comes from contingent valuation studies. These studies typically find that the income elasticity of willingness to pay is less than one [9]. However this evidence was challenged by McFadden and Leonard [14] and McFadden [13], in part on the basis that income elasticities less than one do not accord with economic intuition.

When discussing the distributional impacts of environmental policies, it is important to distinguish between the income elasticity of *demand* and the income elasticity of environmental values. The income elasticities of environmental values are the appropriate concept for understanding the distributional impacts of policies that affect quantity-rationed collective goods [20]. Intuitively there is no reason a good with an income elasticity of demand greater than one (a luxury) may not have an income elasticity of willingness to pay that is substantially less than one. The reverse can also hold true, whereby environmental projects exhibit larger income elasticities for willingness to pay than for demand. Understanding this distinction is the first step in resolving the debate over the distributional impacts of environmental policies.

The central objective of our paper is to investigate and identify both the similarities and differences of these two income elasticities. Hanemann [8] derived the income elasticity of the virtual price for the case of a single public good. This paper draws on those results to generate insights into the debate over the distributional impacts of environmental projects and extends Hanemann's analysis in two directions that are of empirical importance. In Section II, the income elasticities of virtual prices are analyzed when there are multiple public goods. Admitting multiple public goods which are complements can potentially drive a non-trivial wedge between the income elasticities of demands and virtual prices. In Section III, the income elasticities of the virtual prices are extended to analyze the income elasticity of willingness to pay for discrete, as opposed to infinitesimal, changes in environmental quality.

II. THE INCOME ELASTICITIES OF VIRTUAL PRICES

In our analysis we adopt the standard mixed or rationed model of consumption in which consumers have convex preferences over n market goods, denoted by the n-vector X, and k public/environmental goods, which will be denoted by the k-vector Q. Consumers have freedom of choice over the levels of X but face quantity rationing in the levels of Q. Preferences are represented by an increasing, quasi-concave utility function, $U(X, Q)$, which consumers maximize in X subject to a vector of market prices, p, a budget constraint, $p \cdot X \leq y$, and the level of public goods, Q. The maximization problem generates a vector of Marshallian demands,

[2] Grossman and Krueger [5, 6], Seldon and Song [21], and Shafik [22].

$X^m(p, Q, y)$, which represents the optimal levels of market goods and an indirect utility function $v(p, Q, y) = U(X^m(p, Q, y), Q)$. One can also consider the dual minimization problem in which expenditures on X are minimized subject to a given utility level, market prices, and levels of Q. The expenditure-minimizing bundle is the set of Hicksian (compensated) demands, $X^h(p, Q, U)$, and the analog to the indirect utility function is the expenditure function $e(p, Q, U) = p \cdot X^h(p, Q, U)$.

The virtual price (marginal value) of a public good is simply an inverse demand schedule that depends upon the levels of p, Q, and U when utility is held constant and p, Q, and y when income is held constant.[3] We are interested in the degree to which the inverse demand schedule shifts when income is increased.[4] In contrast, when the income effect of demands is considered, the focus is on how *quantity* adjusts. To derive the virtual price vector for Q, define the virtual consumer problem of choosing both X and Q to maximize $U(X, Q)$ subject to the virtual budget constraint $p \cdot X + p^v \cdot Q \leq e^v$, where $e^v = e(p, Q, U) + p^v \cdot Q$. Similarly, the virtual minimization problem is defined as the problem of minimizing expenditures on both X and Q subject to prices p and p^v and utility level U. The virtual prices would induce choosing the same levels of X and Q as those that result under rationing of Q. Using the relationship between the demands from the virtual utility maximization problem and the rationed solution, define the income elasticity of demand at the point of consumption for q_i as

$$\eta_i^d = \frac{\partial q_i^m(p, p^v, e^v)}{\partial y} \frac{e^v}{q_i^m}. \tag{1}$$

In the case of a single public good, Hanemann [8] defines the income elasticity of the virtual price as the partial derivative of the income-constant virtual price scaled by income over the virtual price. Letting the superscript v denote the virtual price, the income elasticity of virtual price i is[5]

$$\eta_i^v = \frac{\partial p_i^v(p, Q, y)}{\partial y} \frac{y}{p_i^v}. \tag{2}$$

Using the two-public good case as a multi-public good example, we can differentiate $Q = Q^m(p, p^v, e^v)$ to derive the income elasticities of virtual prices in terms of the income elasticities of demand. The virtual price and virtual expenditures are implicit functions of income and, therefore, the implicit function theorem can be used.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial q_1^m}{\partial p_1^v} \dfrac{\partial p_1^v}{\partial y} + \dfrac{\partial q_1^m}{\partial p_2^v} \dfrac{\partial p_2^v}{\partial y} + \dfrac{\partial q_1^m}{\partial y} \left(1 + \dfrac{\partial p_1^v}{\partial y} q_1 + \dfrac{\partial p_2^v}{\partial y} q_2 \right) \\[3mm] \dfrac{\partial q_2^m}{\partial p_1^v} \dfrac{\partial p_1^v}{\partial y} + \dfrac{\partial q_2^m}{\partial p_2^v} \dfrac{\partial p_2^v}{\partial y} + \dfrac{\partial q_2^m}{\partial y} \left(1 + \dfrac{\partial p_1^v}{\partial y} q_1 + \dfrac{\partial p_2^v}{\partial y} q_2 \right) \end{bmatrix} \tag{3}$$

[3]The term virtual price was introduced by Rothbarth [19] and is commonly used in the literature which deals with quantity-rationed goods such as Neary and Roberts [16], Madden [11], or Cornes [2].

[4]In this section, no distinction between income-constant and utility-constant virtual prices is required. In the next section, a distinction between the two virtual prices must be made and requires additional notation.

[5]Randall and Stoll [18] refer to this measure as the price flexibility of income.

Equation (3) can be rearranged and simplified as follows. First factor out the demand income terms, $\partial q_i^m / \partial y$, and subtract from both sides. Second, factor out the virtual price income terms, $\partial p_i^v / \partial y$, and rewrite the remaining demand terms using the Slutsky identity. Third, scale both sides of the equation by income and the inverse of the diagonal matrix with diagonal elements q_i. These operations, taken together, provide the relationship

$$
\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{1}{q_1} & 0 \\ 0 & \dfrac{1}{q_2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial q_1^m}{\partial y} \\ \dfrac{\partial q_2^m}{\partial y} \end{bmatrix} y. \tag{4}
$$

σ_{ij}^d is the compensated, cross-price substitution elasticity of demand for q_i and q_j. Denoting the virtual budget share factor of market goods as $S_X^v = p \cdot X / e^v = y / e^v$ and multiplying by one rewritten as virtual expenditure over virtual expenditure, the right-hand side can be completely converted to elasticities.

$$
\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \eta_1^d \\ \eta_2^d \end{bmatrix} S_X^v \tag{5}
$$

The income elasticity of a given virtual price involves considerably more than just the corresponding demand income elasticity. It involves the income elasticities of demand for all of the other rationed goods, the corresponding cross-price demand substitution elasticities (inverted), and the share of virtual expenditures for the n goods in X. As the formulation suggests, the virtual price income elasticity of any element of q_i may differ substantially from its income elasticity of demand and this divergence may come from any one or combination of three factors: the inclusion of other public goods' income elasticities, pre-multiplication by the inverse substitution matrix, or multiplication by the budget share factor for market goods, S_X^v. The budget share of expenditures on market goods from the virtual minimization/maximization problems is always less than one and may be quite small once all of the public goods that affect individual welfare are considered.[6] Thus, one important source of divergence between the income elasticity of virtual prices and demand is the reduction that occurs from multiplying by the budget share factor.

We now turn to potential differences that result from the overall combination of substitution terms and the income elasticities of demand for each of the quantity-rationed goods. The easiest way to demonstrate the potential differences is by providing two examples using the already familiar, two-good case. Examples require knowledge of the income elasticities of demand, the cross-price elasticities, and the budget share factor. Once all of these elements are specified, we can derive the income elasticities of the virtual prices. First suppose that at the point of

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[6] Krutilla [10], for example, notes that "When the existence of a grand scenic wonder or a unique and fragile ecosystem is involved, its preservation and continued availability are a significant part of the real income of many individuals," and provides "the spiritual descendants of John Muir" as potential examples for whom this is true.

consumption we have the following elasticities and budget share.

$$\eta^d = \begin{bmatrix} 0.5 \\ 1.05 \end{bmatrix}, \qquad \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{12}^d & \sigma_{22}^d \end{bmatrix} = \begin{bmatrix} -0.36 & 0.18 \\ 0.09 & -0.88 \end{bmatrix}, \qquad S_X^v = 0.65 \qquad (6)$$

In this case, the virtual price income elasticity vector is

$$\eta^v = \begin{bmatrix} -0.36 & 0.18 \\ 0.09 & -0.88 \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \\ 1.05 \end{bmatrix} 0.65 = \begin{bmatrix} 1.36 \\ 0.92 \end{bmatrix}. \qquad (7)$$

This example was chosen to emphasize that simultaneously we may have the situation of public good one being a demand necessity and public good two being a demand luxury while the respective virtual prices are classified as a luxury and necessity. A more interesting possibility is to consider what can happen when goods are compensated demand complements, a situation which certainly cannot be ruled out.[7] Consider the following example where the goods are complementary. The elasticities and budget share in the first example are fashioned from the range of elasticities typically observed in empirical studies. The elasticities and budget share in this example are from Deaton [3]; the goods are food and entertainment, respectively.

$$\eta^d = \begin{bmatrix} 0.534 \\ 0.585 \end{bmatrix}, \qquad \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{12}^d & \sigma_{22}^d \end{bmatrix} = \begin{bmatrix} -0.275 & -0.049 \\ -0.424 & -0.588 \end{bmatrix}, \qquad S_X^v = 0.683 \quad (8)$$

$$\eta^v = \begin{bmatrix} -0.275 & -0.049 \\ -0.424 & -0.588 \end{bmatrix}^{-1} \begin{bmatrix} 0.534 \\ 0.585 \end{bmatrix} 0.683 = \begin{bmatrix} 1.38 \\ -0.32 \end{bmatrix} \qquad (9)$$

In this example both goods are demand necessities, yet the income classifications of virtual prices turn out to be a luxury classification for the first good and an inferior classification for the second.[8] These two examples demonstrate that knowledge of goods' demand income classifications alone is uninformative with respect to the income classification of the same goods' virtual prices once rationing occurs. Only if one has additional information on substitution parameters for all rationed goods and the virtual budget share of market goods can inferences be drawn. We now turn to an analysis of the income elasticity of willingness to pay for a discrete change in one of the rationed goods.

III. THE INCOME ELASTICITY OF WILLINGNESS TO PAY

Willingness to pay for a discrete change in q_1 from an initial level q_1^0 to a higher level q_1^1 can be defined using the expenditure function as follows where Q is

[7] For example, a potential pair of complementary environmental goods is the preservation of a tract of land for wildlife habitat and the clean up of an adjacent stream.

[8] The result that a good may be classified as a normal demand yet have an inferior virtual price is not available if one models only a single rationed good. Although the single rationed good case is insightful and technically correct if conditioned on all other public goods being held constant, we feel the best approach to virtual pricing is to consider all goods from which consumers receive utility which clearly extends to more than one good.

partitioned into the first element and the $k - 1$ vector of remaining other public goods.

$$WTP = e(p, q_1^0, Q_{-1}, U) - e(p, q_1^1, Q_{-1}, U) \tag{10}$$

Following Mäler [12], willingness to pay can be rewritten in integral form using the compensated virtual price of q_1, p_1^v.

$$WTP = \int_{q_1^0}^{q_1^1} p_1^v(p, s, Q_{-1}, U)\, ds \tag{11}$$

An important feature of willingness to pay is that it is a utility-constant measure. Differentiating a utility-constant measure with respect to income requires recognition of the fact that expenditures on market goods no longer match initial income once a change in Q occurs as is the case along the integration path in (11). This fact makes it necessary to draw a distinction between the utility-constant and income-constant virtual prices. Hereafter let a plain v superscript denote the utility-constant virtual price, $p^v(p, Q, U)$, and the superscript v/y denote the income-constant virtual price, $p^{v/y}(p, Q, y)$. The differential adjustment for expenditures can be seen as

$$\eta_i^v = \frac{\partial p_i^v(p, Q, U)}{\partial y} \frac{y}{p_i^v}$$

$$= \frac{\partial p_i^{v/y}(p, Q, e(p, Q, U))}{\partial y} \frac{\partial e(p, Q, U)}{\partial U} \frac{\partial v(p, Q^0, y)}{\partial y} \frac{y}{p_i^v}. \tag{12}$$

When $Q = Q^0$, the income elasticity of the utility- and income-constant virtual prices are identical since the marginal effect of utility on expenditures equals the inverse of the marginal effect of income on utility. However once $Q \neq Q^0$, the two elasticities will typically no longer be the same because the marginal effect of utility on expenditures will likely change as Q changes. The incorporation of the effect of these changes into a utility-constant version of (5) is given as

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \eta_1^d \\ \eta_2^d \end{bmatrix} S_X^v \left(\frac{y}{e(p, Q, U)} \right)$$

$$\times \left(\frac{\partial e(p, Q, U)}{\partial U} \right) \left(\frac{\partial v(p, Q^0, y)}{\partial y} \right). \tag{13}$$

The term income divided by expenditures results from the fact that $S_X^v = e(p, Q, U)/e^v$ no longer equals y/e^v. Note that all of the potential factors for differences between the income elasticities of demand and the virtual prices still apply from Section II, but now there is also a utility adjustment factor. Using this adjusted formulation, we can now write the income elasticity of willingness to pay.

$$\eta^{WTP} = \frac{\partial WTP}{\partial y} \frac{y}{WTP} = \frac{1}{WTP} \int_{q_1^0}^{q_1^1} \eta_1^v(p, s, Q_{-1}, U) p^v(p, s, Q_{-1}, U)\, ds \tag{14}$$

Define the minimum and maximum virtual price income elasticities as

$$\eta_h^v = \max\{\eta_1^v(p, s, Q_{-1}, U) : S \in [q_1^0, q_1^1]\},$$

$$\text{and } \eta_l^v = \min\{\eta_1^v(p, s, Q_{-1}, U) : s \in [q_1^0, q_1^1]\}$$

It is straightforward to show that

$$\eta_l^v \leq \eta^{WTP} \leq \eta_h^v. \tag{15}$$

Clearly the bounds in (15) may be very wide or perhaps very narrow, depending upon how $\eta_1^v(p, Q_{-1}, U)$ varies over $[q_1^0, q_1^1]$. To address the question to what degree the expenditure adjustment terms come into play, we provide a single rationed good example. Suppose that preferences are represented by a generalized CES utility function which takes the form[9]

$$v(p, q, y) = T[\tilde{v}(p, q, y), p],$$

where (16)

$$\tilde{v}(p, q, y) = [y^{1-\eta} + K(p)q^{1-\eta}]^{1/(1-\eta)}.^{10}$$

In this case η is the uncompensated virtual price income elasticity which is constant for all levels of income and q. This example is especially useful because the compensated virtual price income elasticity will differ only according to the additional adjustments required by the change in expenditures discussed earlier. Table I provides values for the income elasticity of willingness to pay and the compensated minimum and maximum virtual price income elasticities used in the bounds given above.[11]

For each entry in Table I the ratios of η to η^{WTP} fall within the range of 0.6 to 1.5. With only two exceptions, the ratios of η to η_l and η to η_h fall in the same range which suggests that the virtual price income elasticity for most values of q are reasonably close in magnitude to the income elasticity of willingness to pay. Because of willingness to pay's representation as an integral over the compensated virtual price, one can apply the same reasoning used to explain potential differences between the income elasticities of virtual prices and demand to explain potential differences between the income elasticities of willingness to pay and demand. Returning to the case of multiple rationed goods, without knowledge of the substitution between public goods, all other rationed goods' demand income elasticities, and the virtual budget share of market goods, the income elasticity of demand for good one is uninformative with respect to the income elasticity of willingness to pay. This applies to the extent that the income elasticity of demand is not even sufficient in determining whether willingness to pay is increasing or decreasing in income.

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[9] Hanemann [8] used these preferences in one of his simulations analyzing differences between willingness to pay and willingness to accept.

[10] T is an aggregator function assumed to be homogeneous of degree zero, increasing in this first argument, and non-increasing in the other argument.

[11] The values of η, $K(p)$, and y are the same as those used in Hanemann's Table 1.

TABLE I

η	$K(p)$	y	q^0	q^1	η_l	η^{WTP}	η_h
14	0.95	1	1	3	7.18	10.24	14
1.01	1.4	100	1	3	0.99	1.004	1.01
0.677	8.1	100	1	3	0.677	0.967	3.07
0.677	0.1	100	1	3	0.677	0.680	0.684

IV. CONCLUDING REMARKS

In relation to the distributional effects of environmental protection, our analysis indicates that discussing this issue in terms of the income classification of demands may have little, if any, relevance when quantity rationing, the regime we believe to be most relevant, applies. Our analysis shows that an environmental good that is a luxury demand may have an income elasticity of willingness to pay that is greater than one, less than one, or perhaps even negative.

The relationship between the income elasticities of demand and willingness to pay allows the two measures to diverge on account of one or a combination of several factors. First, a given environmental good's income elasticity of willingness to pay depends upon the demand income elasticities of all other rationed goods. Second, the income elasticity of willingness to pay also depends upon the substitutability between the rationed goods. Third, the income elasticity of willingness to pay also depends upon the budget share factor which is always less than one and is potentially much less than one. Finally, there are expenditure adjustment factors which may also account for divergence. The economic intuition behind the empirical results on income and environmental values found in the existing literature can be expressed simply: the rich man may buy proportionately more loaves of bread than his poorer brother, but this does not imply he is willing to pay proportionately more for the same loaf.

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