

Problem Set I: Comments

1. (a) informative means voting for outcome j if and only if the signal is k .
- (b) Naive means voting for outcome 0 when you get signal 0 if and only if $p > q_i$. You vote for 0 if you get signal 1 if and only if $p < 1 - q_i$.
- (c) If naive voting is informative you need $p > q_i$ for all i .
- (d) For majority rule, you are pivotal if and only if the other votes are split. This means that when you get the signal 0 your posterior probability that the true state is 0 (assuming that the others vote informatively) is $p^2(1-p)/[p^2(1-p) + p(1-p)^2] = p$. Hence your best response is to vote for 0 if and only if $p > q_i$. Informative voting is strategic exactly when naive voting is informative. When you get the signal 1 you place probability $1-p$ on the true state being 0 and vote for 0 only if $-pq_i > -(1-p)(1-q_i)$ or $1-q_i > p$. Hence you need $p > \max\{q_i, 1-q_i\}$ for all i in order for informative voting to be strategic.
- (e) I made a mistake. I wanted to ask you to compute an equilibrium when informative voting is **NOT** strategic. In the case of majority voting, this leads to naive voting being an equilibrium (and naive voting being noninformative). As stated, the answer to (e) is the same as (d).

2. For unanimity rule, you are pivotal if and only if the other votes are for 0. This means that when you get the signal 0 your posterior probability that the state is 0 is $p^3/[p^3 + (1-p)^3]$. So your best response is to vote for 0 if this quantity is less than $q/(1-q)$. This condition is easier to satisfy than the corresponding condition for majority rule. When you get the signal 1, the probability that the true state is zero is $p^2(1-p)/[p^2(1-p) + p(1-p)^3] = p$. So you vote for 1 if and only if $1-p > q$. This condition is harder to satisfy than $p > q$ (since $p > .5$).

The answer to the question about whether informative voting is more likely to be strategic voting under majority rule than unanimity is “it depends.” If q is small, then you want to take action 0 only if you are nearly sure that it is correct. This makes unanimity attractive. The conditions for informative voting under unanimity are weaker in this case. On the other hand, unanimous rule won't be strategic if q is bigger than .5.

3. Under majority rule, you make losses of either q or $1-q$ when your action does not match the state. This happens if at least two of the three signals are misleading, with probability $(1-p)^3 + 3p(1-p)^2$. (This quantity divided by two is the probability that the true state is 1 and at least two signals are zero.)

Hence the utility is $-.5[(1-p)^3 + 3(1-p)^2p]$.

In the case of unanimity, you lose $1-q$ when you incorrectly take action 1. This happens with probability $.5(1-p)^3$. You lose q when you incorrectly take action 0. This happens when the true state is 0 and you receive at least one signal 1: $.5[(1-p)^3 + 3(1-p)^2p + 3(1-p)p^2]$. So the utility is

$$-.5\{[(1-p)^3 + 3(1-p)^2p](1-q) + [(1-p)^3 + 3(1-p)^2p + 3(1-p)p^2]q\}.$$

Note that this quantity (unlike the corresponding quantity for majority rule) depends on q . Intuitively, unanimity is better if q is small. The comparison is not completely straightforward because the formulas above assume that strategic voting is rational under both rules. This requires $p = 1 - q > .5$.

4. Assume that all agents have the same voting preferences if they have the same information and these preferences favor the majority signal. Then it is an equilibrium to tell the truth and vote according to the joint information. Plainly the voting part is a best response. The information revelation part is also optimal because giving your opponents more information will always induce them to vote in the way that you want them to. The congruence of preference assumption will hold if q_i does not depend on i and more generally if the q_i are “close.”

Suppose that q_1 is close to 1 (so that agent 1 only wants action 1 if all three signals favor action one), while q_2 and q_3 are close to .5, so that these agents prefer the action that generates the most favorable signals. If you imagine that agents report and vote honestly, then player 1 will want to deviate and always report signal 0. If it turns out that his signal is 1 and the others report 1, then he'll vote for 1. Otherwise, he'll vote for 0. His lie raises the probability that 0 will be the final outcome.

5. Here is the simple comment. Suppose that $c > 0$ and no other agent is getting information. Suppose further, than all agents vote for the same candidate. (If preferences are homogenous, it makes sense to assume that they support the ex ante preferred candidate.) Gathering information cannot change the outcome, so it is a best response not to gather information and to vote with the majority. This is true even if c is small, the signal is quite accurate, and the cost of error is large. That is, under majority rule there is always an equilibrium in which no one gathers information. Under unanimity, the same argument applies if all agents vote for 1 in equilibrium. (There is no incentive to deviate because candidate one always wins.) On the other hand, it may be the case that there is no equilibrium in which no one gathers information and everyone votes for 0 under unanimity. There may be incentive for an agent to get information in order to veto action 0.

Fairly straightforward “value of information” arguments show that for small c , there exist equilibria in which all agents collect information.

Now imagine that you can adjust the voting procedure so that, for example, the person who acquires information goes first. It is easy to think of situations in which if only one person acquires information, he should be allowed to vote first (or send a signal).