Cheap Talk

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OPTIMAL EX POST EFFICIENT MECHANISM

- Cut off rule qualitatively as before.
- Randomization.
- Agents think that they are decisive with sufficient probability.

PROPERTIES

- No information sharing.
- No committees.

WHAT IS MISSING

- No joint production or complementarity.
- No differences of opinion.

NEW TOPIC

- Focus on differences of opinion.
- Minimal need for information exchange.
- Single decision maker

Significant change of direction.
I hope to develop some ideas about communication and then go back and combine them with decision making.

SIMPLE COMMUNICATION MODEL

- Two agents.
- One (Sender) has private information. The other (Receiver) takes action.
- Nature picks state $\theta \in [0, 1]$ from prior.
- Sender learns θ. Receiver does not.
- Sender sends message $m \in M$ to Receiver.
- Receiver takes action $y \in [0, 1]$.

Preferences

$U^i(\theta, y)$, $i = R, S$.
Note (important): U^i does not depend on m.
Talk is cheap.

ASIDE: OTHER POSSIBLE ASSUMPTIONS

- Standard “Spence" signaling: $U^i(\cdot)$ depends on m. Normally assume single crossing.
- Verifiable information. $M(\theta)$ set of messages available to θ. ($M(\theta) = \{\theta\}$, truth required. $M(\theta) = M$, cheap talk.)

ASSUMPTIONS

Leading example:
$U^R(\theta, y) = -(y - \theta)^2$ and $U^S(\theta, y) = -(y - \theta - b)^2$, $b > 0$.
prior uniform.
y^i solves: $\max U^i(\theta, y)$
Note: $y^S(\theta) > y^R(\theta)$.
(In quadratic example, $y^S(\theta) = \theta + b$ and $y^R(\theta) = \theta$.)

EQUILIBRIUM

Three elements:
$\sigma(m \mid \theta)$, probability that θ sends m.
$y(m)$ is R's response to m.
$\mu(\theta \mid m)$ is R's beliefs about θ given m.
Three conditions
S best responds: m used means it solves:
$\max U^S(\theta, y(m'))$.
R best responds: $y(m)$ solves $\max EU^R(\theta, m) d\mu(\theta \mid m)$.
$\mu(\theta \mid m)$ derived from prior and $\sigma(m \mid \theta)$.

SIMPLIFICATIONS

- Why does R use a pure strategy?
  Concavity.
- What about S? It turns out we can take M to be finite and S's strategy to be pure.

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FACT ABOUT CHEAP TALK MODELS

There is always a “babbling" (non informative) equilibrium.

KEY OBSERVATION

Induced action:
$\{y : y = y(m)$ and m sent with positive probability$\}$.
Observation: Only finite number of actions induced in equilibrium.

WHY?

Intuition: If too many actions, then both R and S can approximate their favorite action.

EQUILIBRIA

Unit interval partitioned.
Types in each partition element send the same message.
R best responds.
Incentive constraints determine edges of partition.

PROPERTIES

There is always an equilibrium with 1 action induced.
If there is an equilibrium with N actions, then there is one with $N-1$.
The equilibrium with most actions induced is “nicer."
The maximal number of actions induced decreases with b.
If $\theta = 0$ prefers pooling to separating, then there is a unique equilibrium.