Problem Set I

This is a computational exercise based on a simple voting model. There are two states of the world, 0 and 1; there are two actions, 0 and 1; there are two signal realizations, 0 and 1. The prior probability that the state of the world is 0 is .5. The utility function of agent i is $u_i(0,0) = u_i(1,1) = 0$, $u_i(1,0) = -q_i$ and $u_i(0,1) = -(1-q_i)$ for $q_i \in (0,1)$. (The first argument in $u_i(\cdot)$ is the action, the second argument is the state.) Each agent receives an independent signal. The signals are identically distributed. The probability that the signal is equal to the true state is p, $p > .5$. The population consists of three individuals.

1. Assume that agents vote for either 0 or 1 and 0 is the outcome if and only if at least two voters vote for 0.

   (a) What constitutes informative voting behavior?

   (b) What constitutes naive voting behavior?

   (c) When is naive voting informative?

   (d) When is informative voting strategic (that is, a Nash equilibrium in undominated strategies)?

   (e) Assume that q_i is independent of i and that informative voting is an a Nash equilibrium. Compute a symmetric Nash equilibrium in undominated strategies.

2. Repeat part (d) of the previous question assuming that the outcome is 0 if and only if all three voters vote for 0. Is informative voting more or less likely to be strategic under unanimity?

3. Repeat part (e) of the first question assuming that the outcome is 0 if and only if all three voters vote for 0. Is the equilibrium better or worse (in terms of expected utility) under unanimity or majority rule?

4. Assume that prior to voting agents engage in a round of communication. Each agent can make an announcement (either zero or one). The agents then vote based on their announcement and their private information. Under what conditions is it the case for all agents to honestly report their signals and then vote sincerely (for their conditionally best action)? Answer the question for both unanimity and majority rule.

5. Now assume that individuals must pay a cost $c > 0$ in order to obtain a signal. That is, individuals first decide whether to purchase a signal. If an individual buys a signal, he pays c and gets the information described above. If he does not buy a signal, then he learns nothing. Consider two different information structures: first, the information acquisition stage is simultaneous and private (so that others do not know whether any other agent has purchased information); second, the information acquisition stage is simultaneous and public (so that each agent knows which of the other agents acquire information). Assume that q_i is independent of i and describe the symmetric equilibria of the game under both voting rules. If possible compare the amount of information acquisition to the socially optimal level.