Economics 205, Fall 2010: Quiz II
September 3. 2010

Instructions. Try to answer all three problems. (Read all of the questions now and start on the ones that seem easiest.) Think before you write. You should be able to do everything without much tedious computation. Make your answers as complete and rigorous as possible: give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing.

1. Decide whether each of the statements below is true. If the statement is true, then prove it. If the statement is false, then give a counterexample. In each part $f : \mathbb{R} \to \mathbb{R}$, is twice continuously differentiable and strictly concave.

   (a) $x = 1$ cannot solve $\min f(x)$ subject to $x \in [0, 1]$.

   (b) If $f'(1) = 0$, then 1 is a local maximum of f.

   (c) There exists no function f (satisfying the assumptions of the problem) such that $f(0) = f(1) = f(2)$.

2. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

   (a) Find the eigenvalues of A.

   (b) Find a maximal cardinality set of linearly independent eigenvectors for A. Associate these eigenvectors with the eigenvalues you found in Part a.

   (c) Is A diagonalizable?

   (d) If the matrix is diagonalizable, find a matrix P such that $A = PDP^{-1}$, where D is diagonal.

   (e) State whether the quadratic form $Q(x) = x^t A x$ is positive (semi-) definite, negative (semi-) definite, or indefinite.

3. Let $w = (1, 4, 0)$ and $v = (1, 0, 2)$.

   (a) Find the equation of the line that passes through the point w in the direction v.

   (b) Find the equation of a hyperplane that contains the point w and contains the line you found in part (a).

   (c) Find an equation of a line that is contained in the hyperplane that you found in part (b), contains the point w, and is orthogonal to the line you found in part (a).