

Economics 205, Fall 2010  
Quiz I

August 27, 2010

**Instructions.** Try to answer all 3 problems. (Read all of the questions now and start on the ones that seem easiest). Make your answers as complete and rigorous as possible. When you compute a derivative say “This step follows from the chain rule” or “Because the derivative of a sum is the sum of the derivatives . . .”. When you take a limit, invoke the necessary results (or give a direct proof with  $\epsilon$ s).

Informal and intuitive arguments are better than nothing.

1. Let  $f$  be a differentiable function. Calculate the derivative of the function  $h$  defined in each of the problems below. If you need additional assumptions, make them explicit:

(a)  $h(x) = \log f(x^2)$

(b)  $h(x) = f(\log x)$

(c)  $h(x) = e^{\log x}$

2. Calculate the limits indicated below.

(a)  $\lim_{n \rightarrow \infty} \frac{n^2 - 1}{3n^3 + 6}$

(b)  $\lim_{x \rightarrow 5} \frac{x^2 - 2}{x + 5}$ .

(c)  $\lim_{x \rightarrow 0^+} x \log x$ .

3. Let  $f : [0, 1] \rightarrow \mathbf{R}$ .

- (a) Prove that if

$$|a - b|^{1/2} \geq |f(a) - f(b)| \text{ for all } a, b \in [0, 1], \quad (1)$$

then  $f$  is continuous on  $(0, 1)$ .

- (b) Give an example of a non-constant function  $f$  that satisfies (??). [Prove that your example satisfies the condition.]