Economics 205, Fall 2008: Quiz 3 (Possible Solutions)

Comments: Question 1 was worth 16 (8 for each part); Question 2 was worth 17; Question 3 was worth 17 (5 for the first two parts and 7 for the third part). 50 possible points, max: 50, min: 25, median: 42.

1. (a) $f(x,y) = x^2+6xy+y^2+4x-8y+12$ so $Df(x,y) = \begin{bmatrix} 2x+6y+4 & 6x+2y-8 \end{bmatrix}$, so the critical point satisfies: $Df(x,y) = 0$ or $(x,y) = (7/4, -5/4)$.

$$D^f(x,y) = \begin{bmatrix} 2 & 6 \\ 6 & 2 \end{bmatrix},$$

which is indefinite (first minor positive, determinant negative – the eigenvalues are -4 and 8).

(b) $f(x,y) = y^2 - xy - x^2 + 4y$ so $Df(x,y) = \begin{bmatrix} -y-2x & 2y-x+4 \end{bmatrix}$, so the critical point satisfies: $Df(x,y) = 0$ or $(x,y) = (4/5, -8/5)$.

$$D^f(x,y) = \begin{bmatrix} -2 & -1 \\ -1 & 2 \end{bmatrix},$$

which is indefinite (first minor negative, determinant negative – the eigenvalues are $\pm\sqrt{5}$.

2. Instead solve: $\max -(x^2+2y^2)$ subject to $3x+2y=4$. First order condition: $-2x = \lambda 3$, $-4y = \lambda 2$. This yields: $x = 12/11, y = 4/11, \lambda = -8/11$. Why is this a solution to the problem? It is the only possible local max or local min. The maximum must exist because the constraint set is closed and the objective function is bounded above.

3. We have three equations and four variables, so it is plausible to expect that we can solve for three of the variables in terms of the fourth. Furthermore, the given point does in fact satisfy the equations. So it remains to see whether the condition of the implicit function theorem is met. I will solve by differentiating "everything in sight." When I do, I get three equations for the derivatives of the unknown functions $x = X(w)$, $y = Y(w)$, and $z = Z(w)$. The equations are:

$$Z(w)Y'(w) + Y(w)Z'(w) = 0,$$
$$Z(w)X'(w) + X(w)Z'(w) = 0,$$

and

$$X'(w) + Y'(w) + Z'(w) = 1.$$

When I plug in the given values of x_0, y_0, z_0, and w_0, then these equations become:

$$2Y'(w_0) + 6Z'(w_0) = 0,$$

$$2X'(w_0) + Z'(w_0) = 0,$$

and

$$X'(w_0) + Y'(w_0) + Z'(w_0) = 1.$$

Solving these equations yields:

$$X'(w_0) = 1/5, Y'(w_0) = 6/5, \text{ and } Z'(w_0) = -2/5.$$