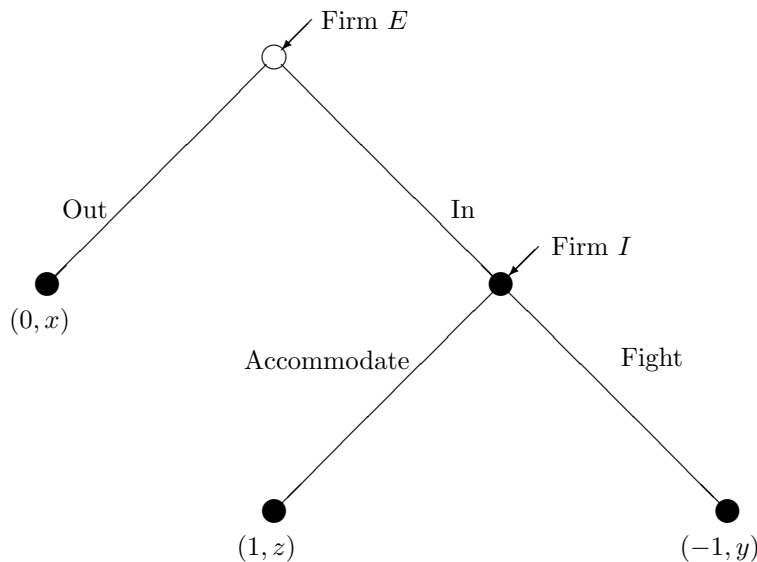


Economics 200C, Spring 2010  
Practice Examination

May 29, 2010

1. At time 0, an incumbent firm (Firm  $I$ ) is already in the market and a potential firm (Firm  $E$ ) is considering entry. In order to enter, Firm  $E$  must incur a cost  $K > 0$ . Firm  $E$ 's only opportunity to enter is at time 0. There are three production periods. In any period in which both firms are active in the market, the game is displayed in the tree below. Firm  $E$  moves first, deciding whether to stay in or exit the market. If it stays in, Firm  $I$  decides whether to fight (the left payoff is for Firm  $E$ ). Once Firm  $E$  plays "out," it is out of the market forever. If Firm  $E$  earns zero in any period during which it is out of the market, and Firm  $I$  earns  $x$ . The discount factor for both firms is  $\delta$ . Assume  $x > z > y$ ;  $y + \delta x > (1 + \delta)z$ ;  $1 + \delta > K$ .



- (a) What is the unique subgame perfect Nash equilibrium of the game?
- (b) Suppose now that Firm  $E$  faces a financial constraint. In particular, if Firm  $I$  fights *once* against Firm  $E$  (in any period), Firm  $E$  will be forced out of the market from that point on. Now what is the unique subgame perfect Nash equilibrium of this game? (If the answer depends on the values of parameters beyond the assumptions above, then explain why.)

2. The questions that follow refer to the repeated Prisoner's Dilemma game with stage-game payoff matrix:

|           |           |        |
|-----------|-----------|--------|
|           | Cooperate | Defect |
| Cooperate | 5, 5      | 0, 8   |
| Defect    | 8, 0      | 1, 1   |

- (a) Assume that the Prisoner's Dilemma is played twice (PD2). That is, players first play the game described in the matrix above, then they learn the outcome, and then they play the game described in the matrix again. Payoffs for PD2 are the sum of the payoffs of the two stages. Describe the normal form of PD2. To do this, state how many pure strategies each player has, what each pure strategy is, and how to obtain payoffs for each strategy profile. It is not necessary to write a payoff matrix (although you may do so if you find it convenient).
- (b) Now consider the infinitely repeated Prisoner's Dilemma (players discount payoffs with the common discount factor  $\delta = .75$ ).
- i. Does there exist a subgame-perfect equilibrium of the game in which both players cooperate in every period on the equilibrium path?
  - ii. Consider the strategy: Begin by cooperating. Cooperate after any history in which there has been no more than one period in which a player defected. In all other histories, defect. Call this strategy "Forgive Once."
    - A. Find a best response to the strategy Forgive Once.
    - B. What is the outcome of the game if player plays Forgive Once and player two plays the best response identified above. Do the strategies Forgive Once and the best response from A constitute a Nash Equilibrium for the repeated Prisoner's Dilemma? If so, prove it. If not, explain why not.
    - C. Is the best response unique? If so, prove it. If not, exhibit another best response.

3. Consider a relationship in which a principal contracts with an agent. The effort of the agent determines the result. Assume that there are three possible states of nature and that the agent can choose between two different effort levels. The table below describes the output for each combination of effort and state:

|        |         | STATE                  |                          |                       |
|--------|---------|------------------------|--------------------------|-----------------------|
|        |         | $\theta = \text{GOOD}$ | $\theta = \text{MIDDLE}$ | $\theta = \text{BAD}$ |
| EFFORT | $e = 4$ | 60,000                 | 30,000                   | 30,000                |
|        | $e = 6$ | 60,000                 | 60,000                   | 30,000                |

The principal and the agent both believe that the three states are equally likely. The objective function for the principal is  $x - w$  and for the agent is  $w^{\frac{1}{2}} - e^2$  where  $w$  is the wage payment to the agent and  $x$  is the output. The principal selects the wage payment; it can depend on output. Assume that the agent's reservation utility is equal to 114. That is, the agent will be willing to work for the principal if and only if the agent's expected utility is at least 114. In parts b and c assume that the agent makes his effort choice knowing the wage function but without knowing the state of nature.

- (a) What would be the effort and the wage in a situation in which the principal could observe the agent's level of effort (and hence could condition his wage payment on effort)?
- (b) Write down the probability distribution that describes the relationship between effort and output. Does this distribution satisfy the monotone likelihood ratio property?
- (c) Assume that the principal cannot directly observe the agent's level of effort.
  - i. What would be the principal's highest payoff if she selects an incentive scheme that induces the agent to take effort  $e = 4$ ?
  - ii. What would be the principal's highest payoff if she selects an incentive scheme that induces the agent to take effort  $e = 6$ ?
  - iii. What effort level does the principal prefer?
- (d) Assume that the principal has the opportunity to hire an expert agent. An expert agent learns the state of nature prior to making his effort choice (but after he agrees to a contract). (So, for example, he can choose effort level 4 in the good state and effort level 6 otherwise.) Assume that the principal cannot directly observe the agent's level of effort.
  - i. What is the principal's profit maximizing incentive scheme when she has an expert agent?
  - ii. Compare the principal's highest payoff with and without an expert agent. Does the principal gain from having an expert agent?

4. A tutor offers to help a student prepare for an exam. If the student does not hire the tutor, then she will fail the class. If she hires the tutor, then the probability she passes the class depends on the quality of the tutor. If she hires a good tutor, she will pass with probability  $\frac{3}{4}$ . If she hires a bad tutor, she will pass with probability  $\frac{1}{4}$ . The student believes that the tutor is equally likely to be good or bad. Assume that the student receives the equivalent of a monetary prize of  $V$  if she passes the exam and 0 if she fails. She seeks to maximize her expected payoff net of any fees. So, if she pays  $x$  to the tutor and expects to pass the class with probability  $p$ , then her expected payoff is  $pV - x$ . The tutor seeks to maximize his expected earnings from tutoring. You may assume that both players use pure strategies.
- (a) Find the (perfect Bayesian) equilibria of the game in which the tutor sets a price, which the student can either accept or reject. (If the student accepts the price  $x$ , then the tutor earns  $x$  and the student earns  $pV - x$ , where  $p$  is the probability of passing. If the student rejects, then both student and tutor earn zero.)
- (b) Find the (perfect Bayesian) equilibria of the game in which the tutor sets a price and also may offer a “double your money back” rebate in case of failure, which the student can either accept or reject. (If the student accepts the price  $x$  with a guarantee, then the tutor earns  $x - 2(1 - p)x$  and the student earns  $p(V - x) + (1 - p)x$ , where  $p$  is the probability of passing. If the student rejects, then both student and tutor earn zero.) Note: the tutor may offer the guarantee, but is not required to do so. Assume that the student cannot deliberately fail the examination.