

Economics 200C: Problem Set II

Due: April 28, 2010

1. Write down the extensive form for the following three-player game. Player I moves first. He can move either down or right. If Player I moves right, then Player II moves (knowing Player I's move) either down or right. If Player II moves right, the game ends and the payoffs are $(1, 1, 0)$. If either of the first two players move down, then Player III moves. Player III is not told which player moved down. Player III can move either left or right. The payoffs are $(3, 0, 3)$ if Player I moves down and Player III moves left; $(0, 3, 0)$ if Player I moves down and Player III moves right; $(3, 0, 0)$ if Player I moves right, Player II moves down, and Player III moves left; and $(0, 3, 3)$ if Player I moves right, Player II moves down, and Player III moves right.

I'm too lazy to draw the picture.

2. Show that there is no Nash equilibrium of the game in Question 1 in which both Player I and Player II move right with positive probability.¹

No matter what Player 3 does at least one of Player 1 or 2 must receive payoff of at least 1.5. So they won't both move right.

3. Find the Nash equilibria of the game in Question 1.

If Player 3 moves left with probability greater than one third, then Player 1 must move down. This creates an equilibrium in which Player 1 moves down, Player 3 moves left, and Player 2 does anything.

If Player 3 moves left with smaller probability, then Player 1 must be moving right with positive probability (else Player 3 would not be best responding) and Player 2 must be moving down with probability one. Hence Player 1 is indifferent, but must move down with probability no greater than one half. There equilibria in which Player 1 mixes equally, Player 2 always goes down, and Player 3 goes left with probability no greater than one third. There are also equilibria in which Player 2 always plays down, Player 3 always plays right, and Player 1 goes right with probability of at least one half.

4. Are there any Nash Equilibria of the game in Question 1 that fail to be subgame perfect? Any that fail to be Bayesian Nash Equilibria? Explain.

No proper subgames, so there are no NE that fail to be subgame perfect. The only proper subform in Player 3's information set, but since that is always reached, weak perfect bayesian does not restrict things. Trembling hand perfect will require Player 2 to move right in the equilibrium in which Player 1 moves down with probability one.

¹Observe that if Player I thinks that Player III will go right and Player II thinks that Player III will go left, then it is a best response for both Player I and III to move right. Furthermore, if both Player I and Player III move right, then neither player learns what Player II actually does. This is what makes the example interesting.

5. Consider a bargaining problem with two parties, a buyer and a seller. The seller can make an item at a cost of c . The item is worth V to the buyer but nothing to the seller. If the seller doesn't make the item, the game ends without trade (players obtain zero payoffs). Otherwise, the buyer takes possession of the item and can either pay a price p or refuse to pay. Assume $V > p > c > 0$ and that the players maximize expected net value.

(a) Find the equilibria of the game (distinguish, if necessary, between subgame perfect and Nash equilibria).

Subgame perfect: buyer doesn't pay, seller doesn't make. The NE outcome is the same, but there are more NE because the buyer can pay with positive probability (as long as p times the probability of payment is less than c).

(b) Modify the game. Suppose at a cost c' the seller can build the item in such a way that she can make it useless to the buyer ("breakable" item). In the new game, first the seller decides whether to make nothing, make the item as in the previous part, or make the item breakable. The buyer, not knowing whether the item is breakable or not, decides whether to buy. The seller then decides (if the item is breakable) whether to break it. Suppose that $c' - c = \epsilon > 0$ and that the seller gets payoff $\epsilon/2$ from breaking the item when the buyer refuses to pay, but payoff $-\epsilon/2$ from breaking the item when the buyer agrees to pay. Find the equilibria.

Working backwards, the seller will break a breakable item if and only if the buyer refuses to pay. When the buyer decides what to do, he compares $V - p$ (what he gets if he pays) to $(1 - q)V$, what he gets if he does not pay (q is the probability that the item is breakable). Hence if $V - p \geq (1 - q)V$ or $q \geq V/p$, then the buyer will be willing to pay. Now we move to the initial choice of the seller. Making the breakable item is never a good idea (it is always cheaper to make it unbreakable as the small reward for punishing a buyer who does not pay is not enough to compensate for the extra cost of making the item breakable). Hence we arrive at the same outcome as before: If the seller makes the item, the buyer must believe that the item is good. Hence the buyer will not pay and hence it is better for the seller not to make the item. The equilibrium outcome is for the seller not to make the item. The most sensible belief-based equilibrium consistent with this outcome involves the buyer thinking that (in the event that he seems the item) the item is unbreakable and refusing to pay, while the seller breaks the breakable item (at the off-the-equilibrium path information set in which the seller produces the item and the buyer refuses to pay).

(c) Repeat the previous part. This time assume that the buyer believes that there is positive probability that the seller's cost of making a

breakable item is less than c . (More precisely, in this game, first nature moves and determines the cost of making a breakable item. The seller, knowing the cost, decides what to do. The buyer, not knowing the cost or whether the item is breakable or not, decides whether to pay.)

As before: Working backwards, the seller will break a breakable item if and only if the buyer refuses to pay. When the buyer decides what to do, he compares $V - p$ (what he gets if he pays) to $(1 - q)V$, what he gets if he does not pay (q is the probability that the item is breakable). Hence if $V - p \geq (1 - q)V$ or $q \geq p/V$, then the buyer will be willing to pay.

Now we move to the initial choice of the seller. This is a bit different. As before, making the breakable item is not a good idea when it is more expensive to make it breakable (it is always cheaper to make it unbreakable as the small reward for punishing a buyer who does not pay is not enough to compensate for the extra cost of making the item breakable). On the other hand, it is better to make it breakable when it is more expensive to make it breakable. What can happen? If the buyer doesn't pay, the seller would not want to sell the item (unless the joy one gets from breaking the item is greater than the cost of production). So the previous equilibrium outcome continues to be an equilibrium outcome. There is another possibility. Imagine that the seller actually produces the item. The seller who likes to make breakable items earns more than the seller who doesn't like to break independent of the buyer's response. This means that there are the following possibilities:

- i. The seller always makes the item, regardless of the relative cost of breaking. This requires it seller earn non-negative expected payoff by making the item. So the probability that the buyer pays is at least c/p . This is possible if the probability that the seller likes to build breakable items is greater than or equal to V/p .
- ii. The seller always makes the item if it is cheaper to make breakable items. When it is costly to make a breakable item, then the seller randomizes. The randomization probability is chosen to make the buyer believe that if the item is produced, the probability it is breakable is exactly V/p . The seller will randomize if the expected profit is zero. So the buyer must pay with probability c/p .