

Economics 200C: Problem Set I

Possible Answers

1. A game has a unique Nash equilibrium in which players use nondegenerate mixed strategies (that is, their strategies place positive probability on more than one pure strategy). Can this game be dominance solvable?

No. If the game is dominance solvable, it has a unique NE. The unique NE must be the unique (pure-strategy) profile that survives the process of iteratively deleting strictly dominated strategies. Because there is a mixed strategy NE, there must be at least two pure strategies that survive this process.

2. (Philip's Theorem) Show that the game:

	Left	Right
Up	a, a	c, d
Down	d, c	b, b

has a pure-strategy Nash equilibrium (no matter what the values of a, b, c, d). If $a \geq c, d$, then $(UP, LEFT)$ is a NE. If $b \geq c, d$, then $(DOWN, RIGHT)$ is a NE. Assume therefore that $\max\{c, d\} > \max\{a, b\}$. Without loss of generality, assume $c \geq d$. Then $d > a$ (else UP is dominant) and hence $b > c$ is needed to rule out $(DOWN, LEFT)$. This is impossible.

3. Given a game $\Gamma(S)$ with strategy set $S = S_1 \times \dots \times S_I$. If $T = T_1 \times \dots \times T_I$ with $T_i \subset S_i$ for each i , then the restriction of $\Gamma(S)$ to T is the new game $\Gamma(T)$ with strategy set T , player set identical to that of $\Gamma(S)$ and payoff functions obtained by restricting the payoff functions. That is, if u_i^S in the payoff function in $\Gamma(S)$, then the payoff function of $\Gamma(T)$ is $u_i^T(t) \equiv u_i^S(t)$. A restriction to T is closed under best responses if for all i and all $t_{-i} \in T_{-i}$ the solutions to:

$$\max u_i(s_i, t_{-i}) \text{ subject to } s_i \in S_i$$

are contained in T_i .

- (a) Show that if the restriction to T is closed under best responses, then any Nash equilibrium of $\Gamma(T)$ is a Nash Equilibrium of $\Gamma(S)$.

This is true by the definition of NE. If t^* is a NE of $\Gamma(T)$, then t_i^* is a BR to t_{-i}^* in T , so t_i^* is a BR to t_{-i}^* in S by closed under BR.

- (b) Show (by example) that if T is not closed under best responses, then there may exist a Nash equilibrium of $\Gamma(T)$ that is not a Nash equilibrium of $\Gamma(S)$.

Take a standard prisoner's dilemma. Let T be the 1×1 game consisting of the dominated strategies of S .

- (c) Must a rationalizable strategy in $\Gamma(T)$ be rationalizable in $\Gamma(S)$? Prove or give a counter example. Would the answer change if T is closed under best responses?

The answer to the first question is no. The example from (b) works.

The answer to the second question is yes. Consider the sets $R_n(G)$ used to define rationalizability (here $G = S$ or T) and the set is the set of strategies that are best responses to strategies in $R_{n-1}(G)$. If $t^* \in R_1(T)$, then it must be in $R_1(S)$ by the closed under best response property. Continue to argue by induction that $R_n(T) \subset R_n(S)$ for all n .

4. I assume that you have access to MWG answers.