Economics 200C, Spring 2010
Midterm Examination

May 3, 2010

IComments. One hundred points possible (first question 5 points each; second question 8 points each; third question 5,5, 7, 7, and 7. Distribution: High 89, Median 58, Low 30. I will not assign letters to the numerical scores, but scores above 80 are fine and those below 50 are not.

I discussed the first question in class.

On the second question, many people answered the first part by saying, roughly: "Trembling hand perfect rules out equilibria in weakly dominated strategies. Strict equilibria avoid weakly dominated strategies. Hence strict equilibria are trembling hand perfect. This logic is fundamentally flawed. Results on the other two parts were better.

On question 3, many people did not confirm the best response property carefully. Some people asserted that subgame perfection implies that in each period players play a NE. Many people did not realize that there are many best responses to strategies in the repeated game.

1. Two people bid simultaneously for a prize. The only possible bids are 10 and 20. Whoever bids highest wins the picture, paying his bid; the other person pays nothing. Ties are broken randomly, with each bidder equally likely to win (and pay). Person i's payoff is zero if he does not win the object. If he wins and pays p_i, his valutation is $v_i - p_i$. Player 1's value, v_1, is commonly known to be 40. Nature choose Player 2's valuation, v_2 . With probability .75 $v_2 = 20$; with probability .25 $v_2 = 60$. Player 2 learns his valuation before making his bid, but Player 1 does not.

   (a) Draw the extensive form of this game, including the move by Nature..

   (b) Draw the normal form of this game.

   Let H and L mean bid High (20) or Low (10) describe Player 1's strategies. Let HH, HL, LH, and LL describe Player 2's strategies (the first letter is what Player 2 does when his valuation is 10, the second is what he does when his valuation is 20).

   In the payoff matrix below the values are expectations. For example, if Player 1 plays L and Player 2 plays LH, then Player 1 half the time when Player 2's valuation is 20. This means that Player 1 wins with probability 3/8. The surplus is 30, so his expected payoff is 90/8. In this case Player 2 wins surplus 40 if his valuation is high and wins surplus 10 half of the time when his valuation is low. So his expected payoff is $40(1/4) + 10(1/2)(3/4) = 55/4$

	HH	HL	LH	LL
H	10, 5	12.5, 0	17.5, 5	20, 0
L	0, 10	3.75, 6.25	11.125, 13.75	15, 10

   (c) Identify any strictly dominated strategies.

   HL and HH are strictly dominated by LH for Player II. L is strictly dominated for Player I.

(d) Identify any weakly dominated strategies.

In addition to the strictly dominated strategies, HH is weakly dominated by LH.

(e) Identify each player's rationalizable strategies.

H for Player 1 (since L is strictly dominated). HL fand HH for Player 2.

(f) Identify all of the Nash equilibria of the game.

(H, HL) and (H, HH) are pure strategy equilibria. All mixtures are also NE.

(g) Identify the proper subgames of the game.

None (at least the way that I write the game).

(h) Identify all subgame perfect equilibria of the game.

Strategies must be as above (because there are no proper subgames).

(i) Identify all of the weak Bayesian perfect equilibria of the game.

Strategies as above. At Player 1's information set, Player one believes that he is at the high valuation ($v_2 = 60$), bid 20, node with probability .25; the high valuation, bid 10 node with probability .75, and the other nodes with probabilities that add to .75 and agree Player II's strategy

2. Consider a game G in which there are I players; the set of pure strategies, S_i, of Player i is finite; and the payoff function of Player i is $u_i(s)$. A strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_I^*)$ is a strict Nash equilibrium if for each i, $u_i(\sigma_i, \sigma_{-i}^*) < u_i(\sigma^*)$ for each $\sigma_i \in \Delta(S_i), \sigma_i \neq \sigma_i^*$.

($\Delta(X)$ is the set of all probability distributions on a set X, so $\Delta(S_i)$ is Player i's set of mixed strategies.)

For each statement below, state whether it is true for all finite games G. If so, prove it. If not, supply a counterexample.

(a) A strict Nash equilibrium must be a trembling-hand perfect equilibrium.

Yes. If s^* is a strict NE, then it must be a (strict) NE of any perturbed game with sufficiently small ϵ. Hence it is the limit of NE of perturbed games.

(b) If σ^* is a strict Nash equilibrium, then each σ_i^* must place positive probability on only one pure strategy. (All strict Nash equilibria are in pure strategies.)

Yes. If the equilibrium places positive probability on two (or more) pure strategies, then each pure strategy in support of the equilibrium strategy must be a best response and so the strict inequality in the definition cannot hold.

(c) If the pure strategy s_i is rationalizable, then it is a best response to a pure strategy.

No.

	A	B	C
H	$10, 0$	$0, 10$	$7, 7$
L	$0, 10$	$10, 0$	$7, 7$

C is not a best response to either H or L, but it is a BR to a 50-50 mixture (so it is rationalizable).

3. Consider the two-player game G with payoff matrix below and $\Gamma(G, .9)$ the infinitely repeated version of G with discount factor $\delta = .9$.

	C	D
C	$7, 7$	$-1, 10$
D	$10, -1$	$0, 0$

(a) Find the minmax point for each player. $(0, 0)$ (by dominance)

(b) Graph the set of feasible, individually rational payoffs of the game.
Convex hull of the four points in payoff matrix intersected with non-negative orthant.

(c) Consider the strategy (in $\Gamma(G, .9)$) of the row player (Player 1):
If $t \neq 3k$ for $k = 1, 2, \ldots$, play D.
If $t = 3k$ for $k = 1, 2 \ldots$ play C if and only if Player 2 played C in period $t - 1$.
Call this strategy s_1^*.

i. Find a best response (in $\Gamma(G, .9)$) to s_1^*. Call this strategy s_2^*.
One strategy is:
If $t \neg 3k - 1$ for $k = 1, 2 \ldots$, play D.
If $t = 3k - 1$ for $k = 1, 2, \ldots$, play C.
Player 2 must play D if $t \neq 3k - 1$ by dominance (his action does not influence what Player 1 does in the future). In the other periods Player 2 can either play D are get payoff 0 or get C and get -1 in the period and 10 is the next period. As long as $\delta > .1$, C is better.

ii. Is (s_1^*, s_2^*) a Nash Equilibrium in $\Gamma(G, .9)$? Is it a subgame perfect Nash Equilibrium? No to both questions. (Player 1 would gain by planing D in the second period.

iii. Is s_2^* the unique best response to s_1^*? Prove or give a counter example.
It is not unique. For example, Player 2 can play D forever in the event that Player 1 plays D in the second period. (The point is that Player 2 can do anything off of the equilibrium path.)