

### Economics 200C: Problem Set IV

Due: April 28, 2009

1. You own a ship. If you can reach your destination, you can sell the contents of the ship for a net profit of  $V$ . Unfortunately, in order to reach your destination, you must pass two points where you may be captured by pirates. If a Pirate  $i$  finds you, he will make a take-it-or-leave-it demand,  $P_i$ . If you accept the demand, you pay the pirate  $P_i$ , and can continue your journey. If not, you return home and have a net return of 0. The pirates try to maximize the amount they earn. (Pirate  $i$  cares only about what he makes.) You try to maximize expected profits (minus payments to pirates).
  - (a) Suppose that the game has perfect information. You know with certainty that Pirate 1 will find you and, if you continue, Pirate 2 will also find you. Everyone know  $V$ ,  $P_2$  knows the demand of  $P_1$  and so on. Draw a game tree. Compute the subgame-perfect Nash equilibrium. Identify at least one Nash equilibrium that is not subgame perfect.
  - (b) Repeat the first part, but this time assume that Pirate  $i$  finds you with probability  $p_i \in (0, 1)$ . Assume that  $p_1$  and  $p_2$  are independently distributed.
2. Nature either \$100 or nothing in a box (with equal probability). Player 1 know what nature did; player 2 does not. The players simultaneously bid for the item, which goes to highest bidder at the highest bid (the loser gets payoff zero; ties are broken randomly). Describe the Nash equilibria of the game.
3. My son Ben is building a guitar. After an internet search, he hired a luthier (maker of stringed instruments), Patrick, to build the neck of the guitar. He paid Patrick \$350 and Patrick promised to deliver the neck in ten weeks. Ten weeks (at this point, ten months) passed and Patrick has not provided the guitar neck.
  - (a) Write down a simple game that explains (or, at least, is consistent with) this outcome.
  - (b) Suppose that Patrick can produce a guitar neck for  $c < 350$  dollars of material and labor. Consider the game in which Patrick first decides whether to build a guitar. If he doesn't the game ends. If he does, then he ships the guitar to Ben, who decides whether to pay the \$350 or not. Analyze this game assuming that the guitar has no salvage value to Patrick.
  - (c) Now imagine that the neck production process can be divided into 350 equal parts. Completing one of these parts costs  $c/350$ . Consider a game in the  $i$ th stage Patrick decides whether to abandon

construction or continue. If he continues, Ben can either pay \$1 or not. If the guitar neck is completed, then Patrick gives it to Ben. Formulate and solve this game.

- (d) Suppose that Ben agrees to send \$350 to a third party, who in turn promises to give the money to Patrick (minus a commission,  $C$ ) provided that Patrick can prove that he delivered the neck to Ben within ten weeks, but otherwise will return the money to Ben (minus the commission). Show that this game has an equilibrium in which Patrick delivers the guitar and he gets paid (assume that  $350 - C > c$ ).
  - (e) (optional) My wife is planning an expensive remodeling project of our house. Do you have any advice for us?
4. Consider the following game. There are two players, a buyer and a seller. First, “nature” moves and place from 1 to 10 apples in a bag (assume that each of the ten outcomes is equally likely). Second, the seller learns the number of apples in the bag. Third, the seller can makes a “sales pitch” to the buyer. The sales pitch includes a take-it-or-leave-it price. If, in the fourth stage, the buyer accepts the price, buyer gets the bag (and its contents) and pays the price and the seller gets the price. If the buyer rejects the price, the payoff to both buyer and seller is zero.
- (a) Find the equilibrium of the game assuming that the seller’s sales pitch is limited to the price (that is, the only thing that the seller can say is “the price for this bag of apples is  $p$ ”).
  - (b) Find the equilibrium of the game assuming that the seller can make sales pitches of the form “there are at least  $k$  apples in the bag” (along with price announcement).
  - (c) Find the equilibrium of the game assuming that the seller can make sales pitches of the form “there are at least  $k$  apples in the bag” (along with price announcement) provided that the statement is true. (That is, the seller can say that there at least  $k$  apples in the bag only if there are at least ten apples in the bag. The difference between this part and the previous one is that in this part the set of available statements depends on nature’s move.)

Be careful to describe the strategy sets and the equilibrium concept that you use. For simplicity, limit attention to pure-strategy equilibria.