Economics 200C: Problem Set IV

Possible Answers

1. If you reach the second pirate, he will ask for V (or V minus the smallest unit) and you must accept. In the first case, this means that you can credibly commit to rejecting all of the demands of the first pirate. There are lots of subgame perfect equilibria. Whenenver the first demand is positive, you must reject it. If the first demand is zero, then you can continue on to you destination (paying your profit to the second pirate). A Nash equilibrium that fails to be subgame perfect is one in which you refuse demands greater than ε. Both pirates ask ε, you pay them and earn $V - 2\varepsilon$.

   In the answer above, I assume that if you pay the first pirate p_1 and then turn back when you reach the second pirate, your payoff is $-p_1$. One could read the question as saying that if you pay p_1 and then turn back your payoff is zero (but Pirate 1 gets p_1). With this interpretation, you must accept all demands $p_2 < V - p_1$ when you reach the final stage. Pirate 2 must ask $p_2 = V - p_1$ when $p_1 < V$ (and anything otherwise). You will certainly receive zero payoff in equilibrium. You are indifferent between accepting any initial offer. The first pirate makes the offer that maximizes p_1 times the probability that p_1 is accepted. (Depending on your strategy, there are many possibilities.) Pirate 2 asks for $V - p_1$ if $p_1 < V$.

   In the second case, the expected value of continuing past the first pirate is $(1 - p_2)V$ (if the second pirate finds you, you lose). You will therefore accept any demands less than or equal to $(1 - p_2)V$ if you meet the first pirate. So Pirate 1 will ask for $(1 - p_2)V$. Your expected payoff is $(1 - p_1)(1-p_2)V$ although ex post you will profit any time you avoid the second pirate.

2. The uninformed bidder cannot use a pure strategy in equilibrium. If he bid $b < 100$, then the informed agent would want to bid "just a bit more" than b, so Player 1's best response would not be defined. If Player 2 bid $b \geq 100$, then he would always win the item when the item is useless. Consequently, payoff would be negative and it would be better to bid 0. The lowest bid in the support of Player 2's mixed strategy must be zero. This is because the informed bidder would never bid less than the lowest bid of Player 2 when the item is valuable and never bid more than the lowest bid otherwise. Hence if Player 2's lowest bid was positive that bid would make negative profits and Player 2 would do better to bid zero. Player 2's mixed strategy also must be supported on an interval. Otherwise, there would be a gap in Player 2's bids. But it would never be a best response for Player 1 make a bid in the gap. Hence Player 2 would do better lowering bids above the gap. Let $F_2(\cdot)$ describe the cumulative distribution of Player 2's mixed strategy. Let $\bar{b}$ be the upper support. In equilibrium, Player 1 will never bid more than $\bar{b}$ (why?). Therefore, if Player 2 makes bids $\bar{b}$, then his payoff will be $.5(100-\bar{b}) + .5(-b)$. That is,

he always wins; half of the time item is valuable. This payoff must equal 0 since the bidder is indifferent between bidding $barb$ and 0. It follows that $\bar{b} = 50$. Now we can look at Player 1's payoff. Player 1 must bid 0 when she learns the item is worthless. Otherwise, a bid b earns $(100 - b)F_2(b)$. This quantity is constant throughout the support of Player 1's mixed strategy. Further, the support of Player 1's strategy must include $\bar{b}$ (if it were higher, then Player 1 could increase profits by avoiding the high bid; if it were lower, then Player 2 wouldn't place positive probability on $\bar{b}$. Hence, since $F_2(\bar{b}) = 1$ and $\bar{b} = 50$, we know that Player 1's equilibrium when the item is worth one hundred is 50 and $(100 - b)F_2(b) = 50$ or $F_2(b) = 50/(100 - b)$. Notice that this distribution is supported on $[0, 50]$ and has an atom at zero ($F_2(0) = .5$). Notice also that we used Player 1's payoffs to characterize Player 2's strategy. This is the way it always is with mixed strategies.

Back to Player 2. He makes $.5(100 - b)F_1(b) - .5b = 0$ so $F_1(b) = b/(100 - b)$. This cdf is also supported on $[0, 50]$, but it has no atoms. You can check that $F_1(b) \leq F_2(b)$. This means that the bid distribution of the informed player stochastically dominates that of the uninformed player (when the information is favorable). That is, the informed bidder bids (stochastically) higher than the uninformed bidder.

3. The easiest game is one in which Ben decides whether to hire (and pay) Patrick and Patrick, conditional on being hired, decides whether to deliver. The payoffs to Ben and Patrick are $(0, 0)$ if Ben decides not to hire; $(V - 350, 350 - c)$ if Ben hires and Patrick delivers; and $(-350, 350)$ if Ben hires and Patrick does not make the neck (V is the value Ben places on the neck). In the firs game, it looks like Patrick behaved "rationally" by keeping the money but not supplying the neck, but Ben was not rational. He should have predicted that Patrick would not provide the neck and saved his money.

   In the second version of the game, Ben will have no reason to pay for the neck once it is delivered. Patrick, foreseeing this, would not produce the neck.

   In the third version, backward induction leads to the same conclusion. Ben won't pay in the final round. Consequently, Patrick will have no incentive to contribute the final increment of effort needed to finish the guitar neck. Hence Ben won't contribute in the penultimate round. And so on.

   When there is a third party, it is an equilibrium for Ben to send the money and for Patrick, knowing that the money will be paid on completion of the project, to supply the neck. If you model this as a game in which Patrick and Ben move simultaneously (unnatural in context), then there is also an equilibrium in which Patrick does not produce and Ben does not pay. This outcome is an equilibrium in the natural sequential version of the game, but it is not subgame perfect. Of course, the existence of a "good" equilibrium depends on the reliability of the third party.

4. The strategy for the seller is an announcement and a price (as a function of the true number of apples). The strategy for the buyer is a decision to accept or reject as a function of what the seller says and does. If the buyer follows the strategy of only accepting prices that are no greater than p^*, then the seller can do no better than offer p^*. This outcome will be an equilibrium in all three parts. These outcomes are weakly perfect Bayesian equilibria in the first two parts. (Following any announcement other than p^*, buyers believe that there are no apples in the bag and refuse to pay anything. Following the announcement of p^*, the buyer believes that there are 5.5 apples (the prior) and provided $p^* \leq 5.5$ would be willing to buy. Notice that the highest price that is accepted (combined with any statement in parts a and b) will be the one asked by the seller no matter how many apples are in the bag. Hence the statements and price can convey no information to the buyer.

   The third part is different. The equilibria above require the buyer to treat all announcements as relatively bad signals about the number of apples available. On the other hand, in part (c) if the seller says: "there are ten apples," then the only "rational" belief is for the buyer to believe that there are 10 apples (even though weak Bayesian equilibrium permits more general beliefs). If we make the restriction that beliefs must be consistent with the information structure (this would be true if, for example, the beliefs can be derived from feasible purely mixed strategies in the game), then the only equilibrium outcome involves the seller making announcements of the form "there are at least k apples in the bag, please pay k" and the buyer accepting.