Economics 200C: Problem Set III Possible Answers

1. You and a friend buy identical objects while vacationing. Both of the items break while being transported back to California. The airline is obliged to pay for damages, but does not know the value of the objects. The airline phones you and your friend and says: "I know that the vase is worth being \$100 and \$200. Tell me exactly how much you paid for it (an integer between 100 and 200). I will pay you the minimum of the claims made by you and your friend. In addition, if one claim is strictly less than the other, I will pay the person who makes the smaller claim an additional \$2." Assume that both you and your friend must answer the airline's question simultaneous (and therefore without knowing the other answer) and seek to maximize the settlement they receive from the airline. Identify rationalizable strategies for this game. Characterize equilibrium outcomes.

   Can \$200 be a best response? If opponent uses a pure strategy, it can be a best response to the bid of \$100. (One can show that the strategy is weakly dominated, but it is not strictly dominated.) Similarly, any bid is a best response if the opponent makes the lowest bid. So all strategies are rationalizable.

   What about NE? Suppose the highest bid in the support of opponent's strategy is $K > 100$. That is, opponent bids K with positive probability, but no higher bid. In this case $K - 1$ is a better response than any higher bid. It yields the same payoff whenever opponent bids less than K and more otherwise. Therefore, there cannot be an equilibrium in which either player bids more than 101 with positive probability. (If one player bid as high as $K > 101$, then the other player would never bid more than $K - 1$, and so K could not be a best response.) If one player bid 101 with positive probability, then it would be a best response for the other to bid 100. Hence one bidder must bid 100. The other bidder best replies with any mixture of 100 and 101 (earning 100). So the equilibria consist of one player bidding 100 and the other mixing between 100 and 101. (The only symmetric pure strategy equilibrium involves both bidding 100.)

2. Consider the game below.

	L	C	R
U	$100, 100$	$-110, 101$	$-110, 101$
M	$101, -110$	$-90, -90$	$-100, -100$
B	$101, -110$	$-100, -100$	$10, 10$

   (a) Show that U and L are strictly dominated.
   C strictly dominates L; M strictly dominates U.

   (b) Find all Nash Equilibria of the game.

With U and L gone, what remains is a 2×2 game. There are three NE. (M, C), (B, R), and a mixed equilibrium in which players play their second strategy with probability 11/12 and their third strategy with probability 1/12.

(c) Assume now that the game is repeated once. That is, players simultaneously select an action (one of the three strategies in the given game); they receive a payoff according to the table above; they choose another action. Payoffs are the sum of the two stage-game payoffs.

i. How many strategies do each player have in this (once-repeated) game?
A strategy consists of what to do in the first period (three choices) and what to do after all nine first period outcomes (three choices each). Therefore there are 3^{10} pure strategies. Notice that when a player makes his first move there are only three possible continuations, so there is a sense in which only 3^4 of these strategies are not redundant.

ii. Construct an equilibrium of the game in which the first period outcome is (U, L). How can this be consistent with part a?
Column player starts with U and plays M unless the first period outcome was (U, L). Row player starts with L and plays C unless first period outcome was (U, L). If the first period outcome was (U, L), then Row plays B and Column plays R is the second period. This strategy profile leads to payoffs $100 + 10$ for both players. Given opponent's strategy, the most that a player can make by deviating in the first period is $101 - 90$. The most that a player can make by deviating in (only) the second period is $100 - 100$.

3. Consider a game in which a monopoly seller has a product that can be produced at zero marginal cost. She faces a continuum of consumers. Consumer v values the item at $\$v$ (v is both the name of the consumer and his valuation). Assume that v is uniformly distributed on $[0, 1]$.

(a) Suppose that the seller sets a single, take-it- or-leave-it price p. Upon seeing the price, each consumer decides (individually) whether to accept or reject. If consumer v accepts, his payoff is $v - p$. If he rejects, it is 0. Describe subgame-perfect equilibria of this game. The seller earns p for each sale that she makes. Are there other equilibria?

Subgame perfection requires that consumers with valuations greater than v accept $p = v$ and those with lower valuations reject this offer. Hence the seller's problem is to pick p to maximize $p(1 - p)$. This leads to price $p = .5$. Consumer v follows the strategy described above (accept prices below v; reject prices above v). The response of consumer v to the price v does not matter. In that sense, there are many subgame-perfect equilibria, but they all look the same.

There are other NE. For example, suppose consumers with $v > .25$ accept prices if and only if they are no greater than .25. The other consumers reject everything. The seller earns nothing when she asks $p > .25$ and earns $.75p$ otherwise. Hence it is optimal for the seller to charge .25. This is not a sensible prediction, because it requires all consumers to coordinate on the dominated action of rejecting attractive prices (that is, if the seller deviated and charged .5, it would be in the interest of half of the consumers to accept this price).

(b) Now suppose that the game lasts two periods. The seller sets two prices, p_1 and p_2. She earns p_1 for each first period sale, and δp_2 for each second period sale ($\delta \in (0,1)$ is the discount factor). Assume first that the seller can commit to setting two prices. That is, she announces p_1 and p_2 in advance. Consumers decide which period in which to buy (each consumer buys at most one item), earning $v - p_1$ for a first period purchase and $\delta(v - p_2)$ for a second-period purchase. Compute a subgame-perfect equilibrium and compare the outcome to your answer in part (a).

In subgame-perfect equilibrium, buyers will buy in the period that gives the greatest surplus. That is, if $v > p_i$ for $i = 1, 2$, consumer v will buy in period one if $v - p_1 > \delta(v - p_2)$ (or $v(1 - \delta) > p_1 - \delta p_2$). The consumer will buy in the second period if the inequality is reversed (and be indifferent under equality). If $v > p_j$ for exactly one j the consumer will buy in period j. If $v < p_i$ for $i = 1, 2$, the consumer won't buy. When seller announces p_1 and p_2, $p_1 \leq p_2$, the buyers divide into two groups. Those with $v > p_1$ will buy in the first period. The others won't buy at all. This reduces to the one-period problem, so the best that the seller can do is to charge $p_1 = .5$. If $p_1 > p_2$ then profits are:

$$p_1(1 - v^*) + \delta p_2(v^* - p_2)$$

where $v^*(1 - \delta) = p_1 - \delta p_2$. You can solve this problem directly, but a clever thing to do is to use the constraint to substitute for v^* are write the objective function (times the constant $1 - \delta$) as

$$p_1(1 - p_1) + \delta p_2(1 - p_2) + \delta(p_2 - p_1).$$

Without constraints, the best thing to do as far as the first two terms are concerned is to set $p_1 = p_2 = .5$. Since we are assuming that $p_2 \leq p_1$, the third term cannot be positive. Hence this is really the solution to the optimization problem.

Substantively: the solution to the problem is to commit to charging the one-period monopoly price.

(c) Continue part (b), but now assume that the seller cannot make commitments. First she announces p_1; then consumers decide whether to buy; then she announces p_2; finally consumers decide whether to accept the second price. Describe subgame-perfect equilibria to this game. Compare the profit to the profit in parts (a) and (b).

The strategy of the consumers is the same as above. One important implication of this is that in the second period, only consumers with valuation below K remain. In the second period, it is a subgame-perfect equilibrium for the seller to charge $p_2(K) = .5K$. Doing so earns profit $.25K^2$. (This is analogous to the first part of the problem.) By backward induction, the equilibrium therefore involves a second-period price of $K/2$ where the consumer with valuation K is indifferent between buying at p_1 or waiting. That is:

$$K - p_1 = \delta(K - K/2)$$

or

$$p_1 = K(1 - \delta/2).$$

Hence the objective function of the seller can be taken to be: pick K to solve:

$$\max(1 - \delta/2)K(1 - K) + \delta(K^2/4).$$

Differentiate this expression to obtain: $K = (2-\delta)/(4-3\delta)$. Hence in equilibrium, the seller starts with the price $p_1^* = (2-\delta)^2/\left(2(4-3\delta)\right)$. Following a price p_1, buyers purchase immediately if and only if $v - p_1 > \delta v/2$ or $v > 2p_1/(2-\delta)$. Second period pricing in optimal given the buyers remaining in the market.

You can check that the seller does not do as well in equilibrium as in the previous parts of the question.

Try to be careful about describing the strategy sets in this problem.

One subtlety is that in theory a v consumer's strategy (and the second period price of the buyer) conditions on the behavior of all other consumers. There are equilibria in which the seller will charge a high price in the second period only if one particular buyer refuses to buy in the first period.

4. Consider a game in which player 2 (the "entrant") first choose in (to enter a market) or out (to not enter). If 2 chooses out, payoffs to players 1 and 2 are $(4, 0)$. If 2 chooses in, player 1 chooses to acquiesce (payoffs $(2, 2)$) or fight (payoffs $(-2, -2)$). Draw the normal and extensive forms for this game. Find all Nash equilibria of the game. Which ones involve dominated strategies? Now suppose Firm 1 plays this game with a sequence of $N - 1$ entrants. The games are played sequentially, so that entrant i gets to see the outcome of the games played with all lower-numbered entrants before playing. Exhibit a Nash equilibrium in which none of the entrants enter. Which (if any) of the strategies in this equilibrium are (strictly or weakly) dominated? Do the same for an equilibrium in which only the final entrant enters. What is the outcome of the iterated removal of weakly dominated strategies in this game?

	IN	OUT
A	$2, 2$	$4, 0$
F	$-2, -2$	$4, 0$

F is weakly dominated. No strategies are strongly dominated. (A, I) is an equilibrium, so is (F, N).

In the N player version, suppose that player 1 always fights (after all histories) and player 2 never enters. This is a NE. It is a NE for player 1 to always fight, except in the final period, and for all but the final entrant to enter. Iterated removal of weakly dominated strategies leads to player one playing A in the final round after all history, which means that the final entrant will enter following all histories; which means that F against the penultimate entrant is weakly dominated; and so on. The only subgame-perfect equilibrium involves entry in all periods and no fighting (after any history).