

Economics 200C: Problem Set II

Due: April 14, 2004

1. Can a player have two strictly dominant strategies? Give an example or prove that this is impossible.
2. Can a strategy be strictly dominated by a non-trivial mixed strategy, but not by a pure strategy? Give an example or prove that this is impossible.
3. Can adding a weakly dominated strategy change the set of Nash equilibria in a game? How about adding a strictly dominated strategy?
4. Ten firms must decide whether to operate at location A or location B . If there are n firms in location A , then each of these firms earns n^2 . If there are m firms at location B , then each of these firms earns $2m^2 - 14$. Describe the pure-strategy Nash equilibria of the game that arises if the firms simultaneously decide upon a choice of location. Write down (but do not solve) an equation that would characterize a symmetric (all firms play the same strategy) mixed-strategy equilibrium for the game. Show (if you can) that this equation has a solution.
5. Three voters ($i = 1, 2, 3$) must decide between two candidates, A and B . The candidate with the most votes wins. Voters 1 and 2 prefer candidate A to candidate B . Voter 3 prefers candidate B . Voters vote simultaneously. Show that there is an equilibrium in which candidate B wins. Show that this outcome disappears if voters avoid weakly dominated strategies.
6. How do risk attitudes determine play in matching pennies? Suppose that a risk-neutral ROW plays matching pennies against an opponent. The ROW player is indifferent between winning receiving nothing (for sure) and the lottery that pays one penny with probability one half and costs one penny with probability one half. The COLUMN player is indifferent between winning K and the lottery that pays one penny with probability one half and costs one penny with probability one half (K may be negative.) Normalize both players' von Neumann-Morgenstern utility function so that the payoff for losing is -1.
 - (a) Compute the payoff for winning as a function of K .
 - (b) Compute the equilibrium of the game for each K .
 - (c) Now suppose that each player can "chicken out." If a player opts out and the other player plays either heads or tails, then the chicken plays the monetary amount c . If both players chicken out, then they each receive the payoff zero. Answer the first two parts (assuming still that ROW is risk neutral).
7. Mixed strategy equilibria arise in games with discontinuous payoffs and continuous strategy sets. For example, consider a game in which an auctioneer "sells" one dollar to the highest bidder. The high bidder wins the dollar, but every agent pays their bid. Concretely, assume that there are two bidders; a strategy for bidder i is a non-negative number b_i . The payoff to bidder i is $\pi_i(b_i, b_j) - b_i$, where

$$\pi_i(b_i, b_j) = \begin{cases} 1 & \text{if } b_i > b_j \\ .5 & \text{if } b_i = b_j \\ -1 & \text{if } b_j < b_i \end{cases}$$

- (a) Find a symmetric equilibrium of this game. [First show that no symmetric, pure-strategy equilibrium exists. Next assume that the strategy is described by a cumulative distribution function $F(\cdot)$ with the

property that if one player bids less than or equal to b with probability $F(b)$, then the other player is indifferent between all bids in the support of $F(\cdot)$. The indifferent condition leads to an equation that you can use to find $F(\cdot)$.

- (b) Are there any asymmetric equilibria of the game (in pure or mixed strategies). Say what you can.