Economics 200C, Spring 2009
Midterm Examination

Instructions. Try to answer all problems. (Read all of the questions now and start on the ones that seem easiest.) Think before you write. Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. (**ALL** questions require justification, but you may use known results provided that you state them properly.) If you are not sure what a question means, please ask me for an explanation. Informal and intuitive arguments are better than nothing.

There are a total of 100 possible points. The table below describes the point values of individual questions.

	Score	Possible
I		30
II		15
III		10
IV		30
V		15
Exam Total		100

1. Consider a two-player game in which player i selects a nonnegative number h_i and the payoff function is given by $\pi_i(h_1, h_2) = 2h_1 + 2h_1h_2 - h_i^2$ for $i = 1$ and 2.

   (a) Prove that this game has no pure-strategy Nash equilibrium.

   (b) Prove that this game has no mixed-strategy Nash equilibrium.

   (c) Assume that strategies are restricted to the interval $[0, M], (M > 0)$. Find the Nash equilibria of the game.

   (d) Assume that strategies are restricted to be the first M positive integers $(M > 1)$. Identify the set of strictly dominated strategies and the set of rationalizable strategies. Is the game dominance solvable?

2. Find the security level for both players in the game depicted below:

	L	R
U	$4, 4$	$5, 7$
D	$7, 0$	$1, 1$

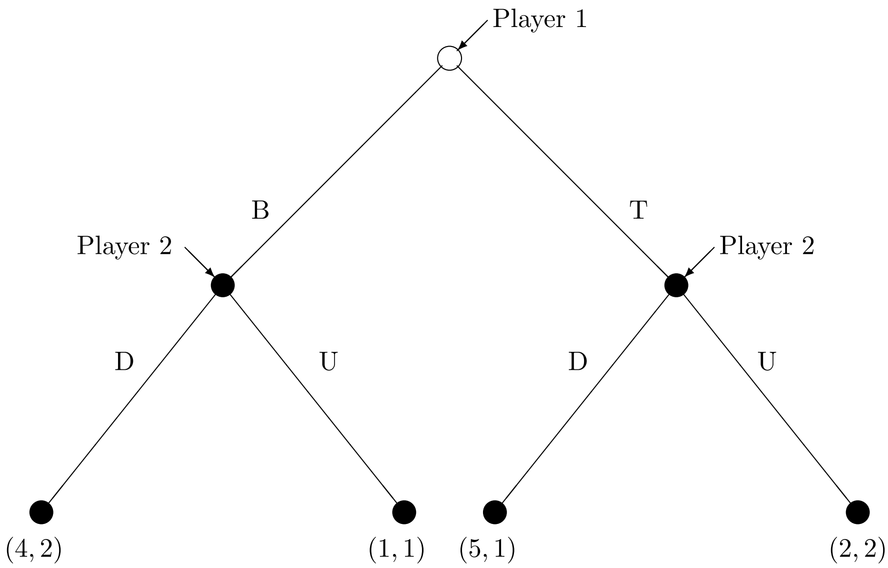
3. Let G be an arbitrary, finite two-player game. (By finite, I mean that each player has a finite set of strategies.) Let G' be the game obtained by adding a strictly dominated strategy for Player 1 to G. Let $\Lambda(G, \delta)$ be the infinitely repeated game derived from G in which players discount payoffs with discount factor $\delta \in (0, 1)$. Let $\Lambda(G', \delta)$ be the analogous repeated game derived from G'. Can the set of Nash equilibrium payoffs of $\Lambda(G', \delta)$ be strictly larger than that of $\Lambda(G, \delta)$? Can the set of subgame perfect Nash equilibrium payoffs of $\Lambda(G', \delta)$ be strictly larger than that of $\Lambda(G, \delta)$? In each case either prove that $\Lambda(G', \delta)$ can be no larger than $\Lambda(G, \delta)$ or provide a counterexample.

4. Consider the extensive-form game described in the diagram below.

(a) Find a subgame perfect Nash equilibrium of this game. Is it unique? Are there any other Nash equilibria?

(b) Now suppose that Player 2 cannot observe Player 1's move. Write down the new extensive form. What is the set of Nash equilibria?

(c) Now suppose that Player 2 observes Player 1's move correctly with probability $p \in (0, 1)$ and incorrectly with probability $1 - p$ (e.g., if Player 1 plays T, Player 2 observes T with probability p and observes B with probability $1 - p$). Suppose that Player 2's propensity to observe incorrectly (i.e., given by the value of p) is common knowledge to the two players. What is the extensive form now? Characterize the pure-strategy equilibria of the game.



5. Decide whether the statements below are true or false. If true, then prove it. If false, give a counterexample. Assume games in parts (a) and (b) have finitely many players and finite sets of pure strategies.

   (a) All Nash equilibria in two-person zero-sum games are Pareto efficient.

   (b) If a game has a unique Nash equilibrium, then it has a unique rationalizable outcome.

   (c) Assume that the payoff matrix below is the stage game for an infinitely repeated game in which both players discount payoffs using the discount factor $\delta = .6$. The repeated game has a subgame-perfect equilibrium in which both players obtain the average payoff 4.

	L	R
U	$4, 4$	$0, 7$
D	$7, 0$	$1, 1$