Economics 200C, Spring 2009
Midterm Examination, Possible Answers

May 6, 2009

Comments on Exam Grading. The grades were in three clusters: seven scores between 80 and 86; five scores between 68 and 72; the remaining scores between 49 and 63 (in this group, 63 is an outlier). The people in the highest group generally know what they are doing. The people in the middle group often know what they are doing. The people in the third group have significant gaps of understanding.

I did the grading. I tried to made comments. Please come to me if you have questions.

If your paper includes a comment of the form: "you may not understand the concept of strategy (or dominance or rationalizability)," I urge you to review the concept.

1. I allocated points: 8, 6, 8, 8. Most got the first and third parts, but perhaps were a bit informal.[1] A significant fraction of people wrote nonsense for part b. The cleanest answer is: since payoff functions are strictly concave in choice variable, best responses are unique, so there cannot be a (non-degenerate) mixed-strategy equilibrium. A large minority of students wrote answers to d that demonstrated lack of understanding of a term (strict dominance, rationalizability, or dominance solvability). Please make sure that you understand that in order for a strategy to be strictly dominated there must exist another strategy that yields a strictly higher payoff no matter what the other players do.

2. The modal answer found the security levels in **pure** strategies. This is not right. I deducted five points.

3. The class did not do well on this question. The fundamental thing to remember is that just because a strategy is dominated in a stage game does not mean a repeated-game strategy that plays the dominated strategy after some histories is strictly dominated. For example, the "tit-for-tat" strategy in the repeated prisoner's dilemma (cooperate originally and immediately following any period in which opponent cooperated; defect otherwise) could be a Nash equilibrium strategy for the repeated game even though cooperate is dominated in the stage game. It is strictly dominated in the repeated game to unconditionally play the stage-game dominated strategy, but otherwise a repeated-game strategy that uses a stage-game dominated action need not be dominated. With this idea in mind, the insight is that adding a dominated strategy for player 1 may lower the minmax payoff of player 2. This could increase the set of individually rational and feasible payoffs and therefore may increase the set of equilibrium average payoffs of the repeated game.

4. This problem came from the text book (I made the last part simpler by limiting attention to pure strategies). Everyone did fine on the first two parts. On the third part there was widespread carelessness about specification of beliefs (a part of the solution). A significant fraction had trouble finding the equilibrium.

[1] Some of the answers to (c) make me think that whoever taught you about constrained optimization didn't do a good job.

5. (a) Notice all outcomes in zero-sum games are efficient. You don't need to use the assumption of NE in your answer, but you do need to use the assumption of zero sum. (b) There was confusion about the meaning of rationalizability. The only problem with (c) is that several people did not describe equilibrium strategies with care.

Comments on First Half Grades As indicated in the class outline, I assigned grades by taking a weighted average of the exam (weight 80%) and the homeworks (20 %). To apply this rule I added up the scores from the homework assignments (10 points for three assignments) to your exam score times 1.2. The maximum number of points is therefore 150. Total scores ranged from 131 to 80. Grade for part 1, G, as a function of total points, X, is:

$$G(X) = \begin{cases} A & \text{if } X \geq 122, \\ A- & \text{if } 122 > X \geq 105, \\ B+ & \text{if } 105 > X \geq 91, \\ B & \text{if } 91 > X \end{cases}$$

ANSWERS

1. The best response of player i to player j's strategy is given by the first-order condition (since the player i's payoff function is strictly concave in player i's strategy). Hence,

$$h_1 = 1 + Eh_2 \text{ and } h_2 = Eh_1$$

where Eh_j is the expectation of h_j (just equal to h_j if player j is using a pure strategy); if Eh_j must exist in equilibrium because otherwise best responses do not exist. From this equation two things follow. First, player i's best response is always a pure strategy (even if the opponent is randomizing). Hence, there is no Nash equilibrium in non-degenerate mixed strategies. Second, if (h_1^*, h_2^*) is a Nash equilibrium, then $h_1^* = h_2^* + 1$ and $h_2^* = h_1^*$. This implies that $h_1^* > h_2^* = h_1^*$, which is impossible. It follows that no Nash equilibrium exists. If strategies must be less than or equal to M, then the best response function for player 2 is the same and for player 1 becomes:

$$h_1 = \min\{1 + Eh_2, M\}.$$

Once again, best replies must be pure strategies. The argument above implies that at least one player must set their strategy equal to M in a Nash equilibrium. However, the formula shows that the best response to $h_j = M$ is to set $h_i = M$. Hence, the unique Nash equilibrium is $h_j = h_i = M$.

If the strategy space consists only of the first M integers, then the game is dominance solvable. Both players playing M is the unique rationalizable outcome. $h_1 = 1$ is strictly dominated for player 1 (because it is never a best response). [You could also point out that for all h_2, $\pi_1(1, h_2) = 1 + 2h_2 < 4h_2 = \pi_1(2, h_2)$, so $h_1 = 2$ strictly dominates $h_1 = 1$.] No other strategies are strictly dominated (they are all best responses to something). Each successive iteration removes the lowest remaining strategy of one player or the other (alternating) until both players have only M remaining.

2. The column player has a dominant strategy (R). Her security level is the worst that can happen if she plays this strategy: 1.

The row player does not have a dominant strategy. One way to compute the security level is to compute the Nash equilibrium of the zero-sum game in which the Row Player's payoffs are as in the game. This involves Row mixing (weight 6/7 on U), leading to a value of 31/7.

3. Adding a strictly dominated stage game strategy can increase the set of subgame-perfect (and hence) Nash Equilibrium payoffs because it makes it possible to increase punishments.

   For example:

	L	R
U	$4, 4$	$0, 5$
D	$5, 0$	$1, 1$

   is a standard prisoner's dilemma. When players are patient any average payoff that is individually rational and feasible is an equilibrium. If we add a strategy for the Row Player that pays -10 to each player independent of Column's action, then Column's security level falls to -10, increasing the set of equilibrium payoffs.

4. (a) The unique subgame perfect equilibrium involves Player 1 moving B; Player 2 moving D if B and U if T. Another Nash Equilibrium involves Player 1 moving T; Player 2 moving U if B (or, more generally, U with probability of at least 2/3) and U if T.

   (b) The new extensive form just puts Player 2's decision nodes into one information set. (T, U) is the only NE (in this case, Player 2 has only two strategies).

   (c) Now Player 2 has two information sets: One after she observes B and one after she observes T. Suppose that Player 1 never plays T in equilibrium. Player 2's action in both information sets must be a best response to B, which is the action D. But then, it is in the interest of Player 1 to play T (earning 5 instead of 4). Consequently, Player 2 must play T with positive probability in equilibrium. Suppose that Player 1 never plays B in equilibrium. Player 2's action in both information sets must be a best response to T, which is the action U. This is an equilibrium. (One can show that there are non-trivial mixed strategy equilibria only when $p = 1/3$ or $p = 2/3$.

5. (a) Yes. All payoffs are efficient in a zero-sum game!

   (b) No. Consider matching pennies.

   (c) Yes. Consider the grim trigger strategy: Play U (L) provided that there has never been a past play of D or R; play D (R) otherwise. For any subgame that follows a deviation, these strategies are in equilibrium (because D and R are stage-game dominant). Otherwise, the best that a deviation can do is to lead to a gain of $3 = 7 - 4$ followed by a loss of 3 in all subsequent periods. Since $3 < 3\delta/(1 - \delta)$ when $\delta = .6$ the statement is true.