Econ 200C, Spring 2012
Problem Set #3
Due: Never, but fair game for exams

1. Consider a signaling model in which firms can observe workers' education levels, but not their productivities, when they decide which workers to hire, and workers, who know their own productivities, choose their education levels. Assume that the output of a worker of type n who has had y years of education is $S(n, y) = nya$, where $0 < a < 1$, and that the cost of y years of education to a worker of type n is $C(n, y) = y/n$. Let $w(y)$ be the wage offered in equilibrium to a worker with y years of education. Assume that $S(\cdot)$, $C(\cdot)$, and $w(\cdot)$ are differentiable.

   (a) Assuming that each worker selects his education level to maximize the difference between his wage and the cost of his education, write the first-order condition that determines how much education a worker of type n chooses to acquire.

   (b) Briefly explain why, in a separating equilibrium, the wage offered a worker with y years of education equals the output of the type of worker who chooses that level of education. (The problem essentially asks you to repeat/recall familiar arguments from class.)

   (c) Using your answers to the first two parts, derive an equation that describes the equilibrium relationship between a worker's wage and her chosen level of education in a separating equilibrium.

   (d) Assuming that $a = 0$ (so education is not productive), determine the level of education chosen by a worker of type n in an equilibrium with parameter K. If n is continuously distributed on $[1, 2]$, for what values of K will your solution from (c) satisfy the nonnegativity constraint on education?

   (e) Indicate which workers would gain, when $a = 0$ (education is abolished).

2. Sue was in a car accident. Roger, the other driver, was responsible. Sue knows the extent of her injuries, t. Roger knows only that t is equal to 10 with probability q and 1 with probability $1 - q$.

   Suppose that Roger has the opportunity to make a take-it-or-leave it settlement offer. If Roger offers s and Sue accepts, then Roger earns $-s$ and Sue earns s. If Roger offers s and Sue rejects, then the case goes to court. Roger earns $-t - c$ and Sue earns t where $c > 0$ are court costs.

   (a) Find the pure strategy equilibria of this game. Does the solution concept matter? Distinguish between Nash, subgame perfect, and perfect Bayesian equilibria.

      Now assume that Sue has the opportunity to make a take-it-or-leave-it settlement demand. If Sue offers x, and Roger accepts, then Sue earns x and Roger $-x$. If Sue offers x and Roger rejects, then Sue earns t and Roger $-t-c$. Assume that Sue can only offer $10+c$ or $1+c$ and that Sue's (type-contingent) strategy is pure but that Roger can randomize in response to a high demand.

(b) Find all equilibria of this game. Distinguish between Nash, subgame perfect, and perfect Bayesian equilibria. Does a pooling equilibrium exist? Does a separating equilibrium exist?

3. David has n items, $n \in \{0, \ldots, N\}$. He can make a take-it-or-leave-it offer to Nageeb. Call the offer p. If Nageeb accepts the offer, then David earns p and Nageeb earns $V_n - p$. Assume that $V_n > V_{n-1}$.

   (a) Find all pure-strategy Nash equilibria of the game in which nature first selects n ($q_n > 0$ is the probability that David is given n items for $n \in \{0, \ldots, N\}$); David learns n; David offers p; Nageeb accepts or rejects p (Nageeb knows the probability distribution q_i, but not n).

   (b) Find all pure-strategy subgame perfect Nash equilibria of the game.

   (c) Suppose now that David can make a statement (an element from a set M) prior to making a price offer. Payoffs are exactly as before. Answer the first two parts.

   (d) Modify the previous question assuming that M consists of all non-empty subsets of $\{0, 1, \ldots, N\}$ and that if David observes n, he can only signal using elements $m \in M$ such that $n \in m$. (That is, David's information is verifiable: he can hide information by sending a "vague" message (m consisting of more than one point), but he cannot lie $n \notin m$).