Econ 200C, Spring 2012
Problem Set #2
Due April 30 (end of clas)

1. Clive owns a car whose value to him is v, where v is uniformly distributed on $[0, 1]$. Rob values the car at $1.5v$. Clive and Rob are both risk neutral, maximizing their expected values of the car and whatever money they receive or pay for it. Clive always knows the actual value of v, but Rob may or may not know v, as indicated below. Except for this, the structure of the game (including the rules discussed below and the fact that Rob's value is 1.5 times Clive's) is common knowledge. Suppose first that Rob knows v, and Rob can make Clive a continuously variable all-or-nothing offer to buy the car at a price p of Rob's choosing, which Clive must either accept or reject. If Clive accepts, then Rob gets the car and pays Clive p; and if Clive rejects, the game ends with no trade.

   (a) Identify the subgame-perfect Nash equilibrium or equilibria of this game. Now suppose that Rob does not know v, and Rob can make Clive a continuously variable all-or-nothing offer to buy the car at a price q of Rob's choosing, which Clive must either accept or reject.

   (b) If Clive accepts an offer of q, what can Rob infer about v from the assumption that Clive is sequentially rational, and thus avoids strategies that do not maximize his expected payoff in every subgame? What is Rob's conditional expected value of v, given q, this inference, and his prior?

   (c) What prices q, if any, is it consistent with expected payoff maximization for Rob to offer, if Clive has a positive probability of accepting Rob's offer?

   (d) Identify the subgame-perfect Nash equilibrium or equilibria of this game, and indicate when the car is sold in your equilibrium or equilibria.

   Finally, suppose that Rob does not know v, and Clive can make Rob a continuously variable all-or-nothing offer to sell the car at a price r of Clive's choosing, which Rob must either accept or reject.

   (e) If Clive offers to sell at price r, what can Rob infer about v from the assumption that Clive avoids strategies that are weakly dominated? What is Rob's conditional expected value of v, given his prior and this inference?

   (f) What prices r, if any, is it consistent with expected payoff maximization for Rob to accept?

   (g) Identify a weak perfect Bayesian equilibrium of this game, and indicate when the car is sold in your equilibrium.

2. Suppose there are two bidders in a first price auction. Each bidder i private observes a random variable t_i that is independently drawn from

a uniform distribution over $[0; 1]$. The actual value of the object to each bidder is $t_1 + t_2$.

(a) Find a symmetric linear equilibrium in which each bidder uses a strategy of the form $s^*(t_i) = \alpha t_i + \beta$. α and β are constants (independent of i).

(b) In the above setting, conditional on only his own private information, t_i, a player's expected valuation for the object is $.5 + t_i$. Consider a private values setting where each player values the object at $.5 + t_i$. The types are drawn as before. Find a symmetric linear equilibrium for this environment.

(c) Show that the equilibrium bid in part (a) is strictly lower than that in part (b) for almost all types of a player. Explain this relationship.

3. Imagine a population of agents indexed by $t \in [a, b] \in [0, 1]$. An agent of type t faces the probability t that he will be in an accident. If an agent is in an accident, his wealth is L. If an agent is not in an accident, his wealth is W. $W > L$. Agents have a common, strictly concave utility function, u defined on wealth. Firms can offer insurance. An insurance policy consists of a price P. An agent who buys an insurance policy agrees to pay P. In exchange, the insurer promises to give the agent $W - L$ in the event of an accident. Consequently an agent who buys insurance receives utility $u(W - P)$, independent of type. An agent of type t who does not buy insurance receives $U(W)(1 - t) + U(L)t$. Agents know their own valuation, but the firms know only the prior. The firm is risk neutral. It earns zero if it sells nothing and if it sells to an agent of type t the firm earns $P - t(W - L)$. $F(t)$ is equal to the fraction of agents with accident probability less than or equal to t. The insurers know F.

   Consider the game in which firms compete by offering insurance at price P (inactive firms earn nothing) and then agents decide whether to buy a policy and, if so, from which firm it wishes to buy.

   (a) Under what conditions does there exist a subgame prefect equilibrium in which the entire market buys insurance?

   (b) Under what conditions there exist a subgame perfect equilibrium in which a non-trivial fraction of the agents buys insurance?

   (c) What would the actuarially fair price for insurance be if everyone were forced to buy insurance? (The actuarially fair price is the price in which insurers make expected profits zero.)

   (d) Suppose now there are two groups (men and women). Insurers can distinguish between men and women (and offer them separate insurance contracts) but cannot directly observe t. Suppose that woman are, on average, healthier than men in the sense that the distribution of accident probabilities for women, F_W is greater than or equal to the distribution of accident probabilities for men, F_M. Compare the equilibrium contracts in this setting to the setting in which it is not possible to distinguish between the two groups. Is it possible for there to be a full-coverage equilibrium with discrimination when such an equilibrium would not exist without discrimination? How about the other way around? Do agents benefit from discrimination? Which ones? How about firms?

(e) Optional policy question: Replace "men" and "women" by "having pre-existing condition" and "healthy record." Comment on the implications of the answer to the previous part.

Try to answer the questions as completely as possible without making parametric assumptions on preferences, W, L, and the distribution of t. You can come close to getting closed-form solutions if you assume that t is distributed uniformly on $[a, b]$, $W = 2$, $L = 1$, and $u(x) = \log x$. (In this case your answers still might depend on the choice of a and b.