Econ 200C, Spring 2012
Problem Set #2
Answer Notes

1. Clive owns a car whose value to him is v, where v is uniformly distributed on $[0, 1]$. Rob values the car at $1.5v$. Clive and Rob are both risk neutral, maximizing their expected values of the car and whatever money they receive or pay for it. Clive always knows the actual value of v, but Rob may or may not know v, as indicated below. Except for this, the structure of the game (including the rules discussed below and the fact that Rob's value is 1.5 times Clive's) is common knowledge. Suppose first that Rob knows v, and Rob can make Clive a continuously variable all-or-nothing offer to buy the car at a price p of Rob's choosing, which Clive must either accept or reject. If Clive accepts, then Rob gets the car and pays Clive p; and if Clive rejects, the game ends with no trade.

   (a) Identify the subgame-perfect Nash equilibrium or equilibria of this game.

   The proper subgames arise for each price offer p. Clive must accept any price $p > v$ and reject any price $p < v$. When $p = v$ any response by Clive is a best response. In equilibrium Rob offer a price that maximizes his payoff given that Clive plays a best response. If Clive accepts with probability one when $p = v$, then Rob's best response is to offer v. Hence one subgame perfect equilibrium is for Rob to offer v and for Rob to accept all prices $p \geq v$ with probability one and reject all $p < v$. This is the only subgame perfect equilibrium because if Clive does not accept $p = v$ with probability one Rob has no best response. (Rob wants to ask $p > v$, but as close as possible to v.)

   Now suppose that Rob does not know v, and Rob can make Clive a continuously variable all-or-nothing offer to buy the car at a price q of Rob's choosing, which Clive must either accept or reject.

   (b) If Clive accepts an offer of q, what can Rob infer about v from the assumption that Clive is sequentially rational, and thus avoids strategies that do not maximize his expected payoff in every subgame? What is Rob's conditional expected value of v, given q, this inference, and his prior?

   Clive accepts q only if $v \leq q$ (and if $v < q$) and rejects it otherwise. Hence Rob can infer that the conditional expected value of v is $q/2$.

   (c) What prices q, if any, is it consistent with expected payoff maximization for Rob to offer, if Clive has a positive probability of accepting Rob's offer?

   If Clive accepts with positive probability, then the conditional expected value of the object to Rob is $3q/4$ and hence Rob's utility is $-q/4$. Rob would therefore make an offer of zero (negative offers, which are always rejected, can also be best response).

(d) Identify the subgame-perfect Nash equilibrium or equilibria of this game, and indicate when the car is sold in your equilibrium or equilibria.

As above: the only equilibria involve non positive prices. Clive's strategy is to accept any price greater than v and reject any price lower than v (how he responds to a price equal to v does not matter.

Finally, suppose that Rob does not know v, and Clive can make Rob a continuously variable all-or-nothing offer to sell the car at a price r of Clive's choosing, which Rob must either accept or reject.

(e) If Clive offers to sell at price r, what can Rob infer about v from the assumption that Clive avoids strategies that are weakly dominated? What is Rob's conditional expected value of v, given his prior and this inference?

It is weakly dominated for Clive to offer $r < v$ because such an offer leaves Clive worse off if Rob accepts it (and the same if Rob rejects it). So when r is offered, Rob can infer that $v \leq r$. If all $v \leq r$ offer r, then it must be that the expected value of v is $r/2$.

(f) What prices r, if any, is it consistent with expected payoff maximization for Rob to accept?

Rob would never accept such a price (because the value to Rob is $.75r$).

(g) Identify a weak perfect Bayesian equilibrium of this game, and indicate when the car is sold in your equilibrium.

The discussion above identifies one equilibrium with no transactions: Rob rejects everything (and Clive offers $r = 1$). This is not the entire story. I assumed that when Clive offers r, then all lower values also do so. A somewhat more subtle version of the argument above demonstrates that if there is an offer accepted with positive probability in equilibrium, there is a smallest (or, to be precise, a greatest lower bound) of such offers. The argument above then shows that such an offer is not accepted.

2. Suppose there are two bidders in a first price auction. Each bidder i private observes a random variable t_i that is independently drawn from a uniform distribution over $[0; 1]$. The actual value of the object to each bidder is $t_1 + t_2$.

(a) Find a symmetric linear equilibrium in which each bidder uses a strategy of the form $s^*(t_i) = \alpha t_i + \beta$. α and β are constants (independent of i).

Suppose that player $j \neq i$ plays $s^*(t_j) = \alpha t_j + \beta$. Suppose that Player i's type is t_i and considers bidding b. The payoff is

$$0 \text{ if } b < \alpha t_j + \beta$$

and

$$t_i + t_j - b$$

otherwise.

Hence the expected payoff is

$$\int_0^{(b-\beta)/\alpha} (t_i + t_j - b)dt_j.$$

Player i now optimizes by maximizing over b. Setting the derivative equal to zero yields:

$$b = \frac{\alpha t_i - \beta(1-\alpha)}{2\alpha - 1}.$$

For this to be in the form $\alpha t_i + \beta$ we much have $\alpha = 1$ and $\beta = 0$.

(b) In the above setting, conditional on only his own private information, t_i, a player's expected valuation for the object is $.5 + t_i$. Consider a private values setting where each player values the object at $.5 + t_i$. The types are drawn as before. Find a symmetric linear equilibrium for this environment.

Now the expected payoff is

$$\int_0^{(b-\beta)/\alpha} (t_i + .5 - b)dt_j.$$

This leads to a first order condition of the form $b = .5t_i + .25 + .5\beta$. For this to be in the form $\alpha t_i + \beta$ it must be that $\alpha = .5$ and $\beta = .5$.

(c) Show that the equilibrium bid in part (a) is strictly lower than that in part (b) for almost all types of a player. Explain this relationship.

The first part asks us to show that $.5(t_i + 1) > t_i$ for almost all $t_i \in [0, 1]$. The inequality holds for all $t_i \in [0, 1)$.

In part (a) when i wins the item, it is a sign that t_j is low (because j bid less than i, which happens for the lower values of t_j). This is a bad sign for i because the value of the item to i depends on t_i. On the other hand, in part (b) the fact that t_j is low does not reduce the value of the item to i – i just cares about t_i. Hence in (a) there is a "winner's curse" (winning the item is evidence that the other party has information that reflects badly on the value of the item. This is not true in (b). Rational bidders take this information into account. It suppresses bids.

3. Imagine a population of agents indexed by $t \in [a, b] \in [0, 1]$. An agent of type t faces the probability t that he will be in an accident. If an agent is in an accident, his wealth is L. If an agent is not in an accident, his wealth is W. $W > L$. Agents have a common, strictly concave utility function, u defined on wealth. Firms can offer insurance. An insurance

policy consists of a price P. An agent who buys an insurance policy agrees to pay P. In exchange, the insurer promises to give the agent $W-L$ in the event of an accident. Consequently an agent who buys insurance receives utility $u(W-P)$, independent of type. An agent of type t who does not buy insurance receives $U(W)(1-t)+U(L)t$. Agents know their own valuation, but the firms know only the prior. The firm is risk neutral. It earns zero if it sells nothing and if it sells to an agent of type t the firm earns $P-t(W-L)$. $F(t)$ is equal to the fraction of agents with accident probability less than or equal to t. The insurers know F.

Consider the game in which firms compete by offering insurance at price P (inactive firms earn nothing) and then agents decide whether to buy a policy and, if so, from which firm it wishes to buy.

(a) Under what conditions does there exist a subgame prefect equilibrium in which the entire market buys insurance?

If everyone agent buys insurance, then $U(W-P) \geq u(W)(1-t) + U(L)t$ for all t. The constraint is tighter when t is smaller, so you need $U(W-P) \geq u(W)(1-a)+U(L)a$. Also, in equilibrium the firms made zero profit. Hence $P = \bar{t}(W-L)$, where $\bar{t}$ is the mean of t (according to F). So one inequality (in terms of the given information W, L, F, u determines whether the entire-market equilibrium exists. Notice that this is impossible if $a = 0$.

(b) Under what conditions there exist a subgame perfect equilibrium in which a non-trivial fraction of the agents buys insurance?

I hope that it is clear that one will have everyone type above a cutoff type buying insurance. If c is the cut off and $t(c)$ is the expected value of t given $t > c$, then we need:

$$U(W-P(c)) = u(W)(1-c)+U(L)c$$

where $P(c) = t(c)(W-L)$.

There will be an interior solution to these equations if $U(W-P(a)) < u(W)(1-a)+U(L)a$ and $U(W-P(b)) > u(W)(1-b)+U(L)b$, which in turn will hold (for example) if a is close to zero and b is close to one.

(c) What would the actuarially fair price for insurance be if everyone were forced to buy insurance? (The actuarially fair price is the price in which insurers make expected profits zero.)

We needed to compute this in the first part: $P = \bar{t}(W-L)$

(d) Suppose now there are two groups (men and women). Insurers can distinguish between men and women (and offer them separate insurance contracts) but cannot directly observe t. Suppose that woman are, on average, healthier than men in the sense that the distribution of accident probabilities for women, F_W is greater than or equal to the distribution of accident probabilities for men, F_M. Compare the equilibrium contracts in this setting to the setting in which it is not possible to distinguish between the two groups. Is it possible for there to be a full-coverage equilibrium with discrimination when such an equilibrium would not exist without discrimination? How about the other way around? Do agents benefit from discrimination? Which ones? How about firms?

Assuming that the support of the distributions is the same, the problem essentially asked you to compare the solution to $U(W - P_i(c)) \geq u(W)(1-c) + U(L)c$ where $P_i(\cdot)$ is the price function derived from the appropriate distribution (men alone, women alone, and mixed). I will assume that the price function for women will be lower than that for men (that is, the conditional mean of t given that $t > c$ is lower for women for all c).

For a full coverage equilibrium, the women would have the lowest price. So if there is a full coverage equilibrium for the men or for the whole group, there must be one for the women. It is possible for there to be a full coverage equilibrium for women, but not for the entire group. In full coverage equilibrium, women benefit from being separated from men. Men benefit from being mixed with women. When there is an interior equilibrium, figuring out the trade off requires a bit more work. You can ask how does the equilibrium cutoff changes when the distribution of risks shift. You can check that there always is a higher participation equilibrium equilibrium in the female market. This equilibrium will have a lower price for two reasons: first, the risks are lower for each cut off (by assumption), second, if more people get insurance, the pool is healthier. Hence women pay less if they buy insurance and more of them buy. This is good. It would be beneficial for a male to be pooled with the women and bad for a woman to be pooled with men. Finally, the firms always make zero profit in this model so discrimination does not matter.

(e) Optional policy question: Replace "men" and "women" by "having pre-existing condition" and "healthy record." Comment on the implications of the answer to the previous part.

Try to answer the questions as completely as possible without making parametric assumptions on preferences, W, L, and the distribution of t. You can come close to getting closed-form solutions if you assume that t is distributed uniformly on $[a, b]$, $W = 2$, $L = 1$, and $u(x) = \log x$. (In this case your answers still might depend on the choice of a and b.)