

Econ 200C, Spring 2012
 Problem Set #1
 Due April 16

I will clarify in class which problems have a positive probability of being graded.

1. Nageeb's notes contain many problems. The Chapter 1 problems range require fairly simple manipulations of definitions and routine computations. (That does not mean that they are easy, but they are not hard when/if you are comfortable with basic definitions and logical arguments.) The Chapter 3 problems are more involved. I recommend 19-21.
2. Variations on min max. These problems require patience and an understanding of the definitions.

Consider the following quantities ($\Delta(X)$ is the set of all probability distributions on X):

$$\max_{\sigma_i \in \Delta S_i} \min_{\sigma_{-i} \in \Delta S_{-i}} u_i(s_i, \sigma_{-i}) \quad (1)$$

and

$$\min_{\sigma_{-i} \in \Delta S_{-i}} \max_{\sigma_i \in \Delta S_i} u_i(s_i, \sigma_{-i}) \quad (2)$$

- (a) What is the relationship between (1) and (2)?
- (b) How do (1) and (2) if you replace (a) $\Delta(S_i)$ by S_i ; (b) $\Delta(S_{-i})$ by S_{-i} ; (c) $\Delta(S_{-i})$ by $\prod_{j \neq i} \Delta(S_j)$? (Answer each part independently.)
3. The purpose of this exercise is to get you to think about the relationship between a dominated strategy in a stage game and a dominated strategy in a repeated game. Given a game G , form a new game G' by adding a strictly dominated strategy for Player 1 such that the infinitely repeated game derived from G' has a strictly larger set of subgame perfect equilibria than the repeated game derived from G when the discount factor is sufficiently close to one. Does the repeated game derived from G' have any strictly dominated strategies? If so, find one (and prove that it is strictly dominated). If not, explain why not.
4. The first two parts of this question should be straightforward. The final part requires a bit of thought.

The questions that follow refer to the repeated Prisoner's Dilemma game with stage-game payoff matrix:

	Cooperate	Defect
Cooperate	5, 5	0, 8
Defect	8, 0	1, 1

I will call this game G .

- (a) Find the smallest $\delta_0 \in (0,1)$ such that for all $\delta > \delta_0$, there exists a subgame perfect equilibrium to the infinitely repeated game with discount factor δ that gives each player average payoff 5.
- (b) How would your answer to part a change if you replaced “subgame perfect” by “Nash”?
- (c) Suppose that there is a population of $2N$ players. N players always play Row; N players always play Column. In each period, Row and Column players are paired and each of the N pairs play G . After they play G , they are paired with another player and the process continues. Assume that pairing is deterministic (each Row player cycles through the N Column players in a fixed order, and similarly for the Column players). This is the extended game.
 - i. Suppose that players can perfectly observe all past plays. Is it possible to construct a subgame perfect equilibrium of the extended game that gives each player average payoff 5? If so, can this be done for any $\delta > \delta_0$?
 - ii. Repeat the previous part assuming that players can observe only the outcome of stage games in which they played. (That is, they do not observe the outcome of a stage game played by a distinct pair of players.)

As part of your answer to part (c) explicitly consider the strategies “cooperate until any opponent defects, defect after any history that contains a defection” and “cooperate with opponent i provided that opponent i has always cooperated with you, defect against opponent i after any history in which opponent i has defected against you.

Do your answers change if you consider Nash rather than subgame perfect equilibrium?