Economics 200C, Spring 2012
Practice Midterm

I tried to write questions that are in the same form, the same length, and the same difficulty as the actual exam questions. I failed. I think that the real exam will have more questions and the questions will need shorter, simpler answers. Rough guidelines: You are responsible for "everything," but I really want to make sure that you know the main ideas. Some of the main ideas are working with and manipulating definitions. Do you know what a strategy is? Can you go from strategy to outcome? Can you identify a best response? Can you identify an equilibrium? Do you know the difference between Nash and subgame perfect? What is the single-crossing condition? What is the difference between signaling and screening? Some of the main ideas involve understanding basic results. In my opinion, the main results are: the folk theorem of repeated games, the possibility of market failure with adverse selection, the structure of equilibria in signaling games (pooling versus separating), the efficiency properties of signaling equilibria, the non-existence of equilibrium in screening models. The questions below do not cover all of these topics, but combined with problems from the text and homework problems, I hope that you will be prepared for Wednesday's test.

1. The questions that follow refer to the repeated Prisoner's Dilemma game with stage-game payoff matrix:

	Cooperate	Defect
Cooperate	$5, 5$	$0, 8$
Defect	$8, 0$	$1, 1$

Consider the infinitely repeated Prisoner's Dilemma (players discount payoffs with the common discount factor $\delta = .75$).

(a) Does there exist a subgame-perfect equilibrium of the game in which both players cooperate in every period on the equilibrium path?

(b) Consider the strategy: Begin by cooperating. Cooperate after any history in which there has been no more than one period in which a player defected. In all other histories, defect. Call this strategy "Forgive Once."

i. Find a best response to the strategy Forgive Once. Is your strategy a best response to Forgive Once in every subgame? If so, find a best response that is not a best response to Forgive Once in every subgame. If not, find a best response that is a best response to Forgive Once in every subgame.

ii. What is the outcome of the game if player plays Forgive Once and player two plays the best response identified above. Do the strategies Forgive Once and the best response from (i) constitute a Nash Equilibrium for the repeated Prisoner's Dilemma? If so, prove it. If not, explain why not.

iii. Is the best response unique? If so, prove it. If not, exhibit another best response.

(c) Suppose agents play the repeated prisoner's dilemma, but they do not observe the outcome of the stage game perfectly. Specifically, assume that if the stage game outcome is (s_1, s_2), then players observe (s_1, s_2) with probability γ and each of the other three outcomes with equal probability $(1 - \gamma)/3$. Suppose both players play Forgive Once (Begin by cooperating. Cooperate after any history in which there has been no more than one period in which (C, C) has been observed). What is the expected payoff if both players play Forgive Once?

2. A tutor offers to help a student prepare for an exam. If the student does not hire the tutor, then she will fail the class. If she hires the tutor, then the probability she passes the class depends on the quality of the tutor. If she hires a good tutor, she will pass with probability $\frac{3}{4}$. If she hires a bad tutor, she will pass with probability $\frac{1}{4}$. The student believes that the tutor is equally likely to be good or bad. Assume that the student receives the equivalent of a monetary prize of V if she passes the exam and 0 if she fails. She seeks to maximizes her expected payoff net of any fees. So, if she pays x to the tutor and expects to pass the class with probability p, then her expected payoff is $pV - x$. The tutor seeks to maximize his expected earnings from tutoring. You may assume that both players use pure strategies.

   (a) Find the (perfect Bayesian) equilibria of the game in which the tutor sets a price, which the student can either accept or reject. (If the student accepts the price x, then the tutor earns x and the student earns $pV - x$, where p is the probability of passing. If the student rejects, then both student and tutor earn zero.)

   (b) Find the (perfect Bayesian) equilibria of the game in which the tutor sets a price and also may offer a "double your money back" rebate in case of failure, which the student can either accept or reject. (If the student accepts the price x with a guarantee, then the tutor earns $x - 2(1-p)x$ and the student earns $p(V - x) + (1-p)x$, where p is the probability of passing. If the student rejects, then both student and tutor earn zero.) Note: the tutor may offer the guarantee, but is not required to do so. Assume that the student cannot deliberately fail the examination.

3. Consider a labor market in which there is one worker and two potential employers. The worker is described by one characteristic, height. Height h can take on one of two values, tall ($h = 1$) or short ($h = 2$). Each worker has a reservation utility r_h that depends height. Assume that $r_h = \alpha_h$, where $0 < \alpha_1 < \alpha_2 < 1$. If a firm employs the worker at wage w, the firm earns $h - w$ when the worker's height is h. If a firm does not employ the worker, the firm earns 0. If a worker accepts a job with wage w, the worker earns w. If the worker does not accept a job, the worker earns r_h. The game works like this. First, nature picks h. Second, the worker learns h. Third, Firm 1, which conducts in-person interviews, learns h. Firm 2 does not learn h. Fourth, Firms 1 and 2 simultaneously make wage offers to the worker. Fifth, the worker selects one of the job offers or rejects both. Firms 1 and 2 are different, therefore, because Firm 1 can condition its offer on the worker's height and Firm 2 cannot.

   (a) If Firm 1 offers 0 wages (to both tall and short workers), what is Firm 2's best response (assuming that the worker responds by maximizing her payoffs).

   (b) If Firm 2 offers the wage 0, what is Firm 1's best response (assuming that the worker responds by maximizing her payoffs)?

   (c) Prove that, in any subgame perfect equilibrium, Firm 2 makes zero profits.

   (d) Prove that, in any subgame perfect equilibrium, Firm 2 never hires short workers.

   (e) Describe the subgame perfect equilibria of the game.