Econ 200C

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Signaling

1. Two players, S (sender) and R (receiver)
2. Nature picks t type of sender. $p(t)$ is probability that type is t.
3. Sender observes t, selects signal, s.
4. Receiver observes s (but not t), selects action a.
5. $U_i(a, t, s)$ payoff function.

Standard application: S is worker, t is ability, s is education, R is market wage. Possible preferences: $U_R(a, t, s) = -(a - t)^2$ (market pays wage equal to expected ability)
$U_S(a, t, s) = a - \alpha s^2/t$ (workers like higher wages and lower signals, marginal cost of producing signal decreases with ability).

Basic Question

Is it possible for the signal to convey information to the receiver?
Answer: Maybe not.
Suppose that $s(t) \equiv s^*$.
The best response for the Receiver includes
$a(s^*) = arg \max EU_R(a, t, s^*)$ (prior optimal action).
If one can find $a(s)$ for $s \neq s^*$ such that
$U_S(a^*(s^*), t, s^*) \geq U_S(a(s), t, s)$ for all t and $s \neq s^*$, then there is a pooling equilibrium outcome.
Finding such an $a(\cdot)$ is not hard in leading examples (in the labor market, let $a(s) \equiv 0$, so that in a putative pooling equilibrium, agents get the average wage for signal s^* and zero otherwise.

Definition of Equilibrium

1. Sender strategy: $\sigma(t)$ mapping type to signal.
2. Receiver strategy: $\alpha(s)$ mapping signal to action.
3. Receiver belief: $\mu(t \mid s)$ updating beliefs given signal.

$(\sigma^*, \alpha^*, \mu^*)$ is a (weak perfect Bayesian) equilibrium if:

1. $\sigma^*(t)$ solves $\max_s U^S(\alpha^*(s), t, s)$ all t.
2. $\alpha^*(s)$ solves $\max_a EU^R(a, t, s) d\mu(t \mid s)$ all s.
3. μ^* derives from prior and σ^* using Bayes's Rule (whenever possible).

Single-Crossing Condition

If $t' > t$, and $s' > s$, then $U_S(a', t, s') = U_S(a, t, s)$ implies that $U_S(a', t', s') > U_S(a, t', s)$.
This is the fundamental sorting condition that arises in many applications of information economics.
Geometrically it states that indifference curves in (a, s) space of different types cross at most once.
Mathematically it can be thought of as a supermodularity assumption on utility.
Economically, it says that higher types are more willing to use higher signals.