Econ 200C

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Incomplete Information

The applications all involve models of incomplete information.

We can study these games using the techniques developed using a "trick" of Harsanyi.

Assume that there is an underlying "type" space that summarizes all of the uncertainty.

Player i's type determines his payoff function and possibly other characteristics.

Assume now that the game is one of imperfect information. It begins with a move of nature – selecting players' types. Nature then tells each player his or her type (but not the types of others).

Solve using "standard" techniques.

Formality

Bayesian Game: $\{I, \{S_i\}, \{u_i\}, \{T_i\}, F\}$, $S = \Pi_{i=1}^{I} S_i$, $T = \Pi_{i=1}^{I} T_i$

- I – player set.
- S_i – strategy set of player i.
- $u_i : S \times T \to \mathbb{R}$ – payoffs
- T type space (summarizes private information).
- F distribution on T.

Equilibrium is a profile of $s_i^* : T_i \to S_i$ such that for each i and $t_i \in T_i$

$$s_i^*(t_i) \text{ solves } \max_{s_i(t_i) \in S_i} Eu_i(s_i, s_{-i}^*(t_{-i}), t) dF(t \mid t_i).$$

Note: You need a specification of the strategy for all types (even though only one type is realized). It does make sense for u_i to depend on the entire vector t.

Adverse Selection

Standard Example: You are trying to buy a used car. The owner of the car knows more about the car's quality than you do. If there is a "market price" then only "bad" cars will be offered for sale.

Competitive Market

1. Given set of types, $t \in T$ ($T = [0,1]$ will be standard).
2. $F(\cdot)$ prior on types.
3. $r(t)$ reservation wage; non-decreasing.
4. t market value of type t's product.
5. w wage.

Market outcome: $\{t : w \geq r(t)\}$ transact at w.
Demand: Infinite, any, zero depending on whether expected productivity is greater than, equal to, or less than wage.

Interpretation

t is marginal product of a worker (the value of worker to employer)
$r(t)$ is reservation wage (what worker can get if not employed)
w is market wage of employed worker.
One imagines that a market equilibrium involves a price with the property that (a) wage is equal to average quality of employed worker; (b) workers are employed if and only if wage exceeds reservation wage; (c) markets clear.

Competitive Equilibrium

A competitive equilibrium is a wage w^* and a set of types T^* such that:

1. $T^* = \{t : r(t) \leq w^*\}$
2. $w^* = E[t \mid t \in T^*]$

T^* is the set of active workers. The first condition says that workers behave rationally: They are active if and only if their reservation wage is less than the market wage w^*.
The second condition says that firms earn zero profits (so it is a market-clearing condition).
Notice that the second condition does not make sense if T^* is empty.
If T^* is empty, then (2) places no constraints on w^*.
(So there will always be an "inactive" equilibrium.)

Market Failure

There may be no competitive equilibrium with active traders even when there are always gains from trade. This is called adverse selection.

Suppose that $r(t) = \alpha t$ for $\alpha \in (0, 1)$. In this case, every worker is more valuable in the market than outside. Suppose that t is uniformly distributed on $[0, 1]$.

What happens if the wage is w?

All workers with $\alpha t \leq w$ enter the market. So the quality of a worker is uniformly distributed on $[0, w/\alpha]$ and the average quality is $w/2\alpha$.

An interior equilibrium requires that $w^* = w^*/2\alpha$ or $\alpha = .5$. If $\alpha > .5$ the market cannot be active since the average quality of worker is less than the wage.

(When $\alpha < .5$ all workers are active.)

Strategic Model of Adverse Selection

The competitive model does not allow firms to compete – they take wages as given.
Does this matter?

- Nature picks types according to distribution.
- Workers learn type, but firms do not.
- Firms (assume that there are two) simultaneously make wage offers.
- Workers select a firm to work for (or opt out of the market).
- Preferences as before.

Analysis

Informally: The equilibrium outcome is the highest competitive market wage.
More formally: Let $\bar{w}$ be the highest competitive market wage and let $\underline{t}$ be the lowest type.
Either: $\bar{w} = r(\underline{t})$, in which case the market shuts down (wages are not determined in equilibrium, but they must be no higher than $\bar{w}$)
or: There is a unique subgame perfect equilibrium in which firms offer $\bar{w}$ provided a regularity condition holds. A sufficient condition for the result is that $E[t \mid r(t) \leq w] > w$ for w slightly less than $\bar{w}$.
The regularity condition says that you cannot have an equilibrium with wages slightly below $\bar{w}$.
The condition will be satisfied if $E[t \mid r(t) \leq w] - w$ is strictly decreasing at $\bar{w}$. (We know that it is weakly decreasing.)

Interpretation

Adding strategic power rules out low activity (low wage) equilibria, but does not eliminate adverse selection.

Argument

If $\bar{w} = r(\underline{t})$, then $E[t \mid r(t) \leq w] < w$ for all w, so no firm will offer more than $\bar{w}$ (else it will make negative profits).
If $\bar{w} > r(\underline{t})$,

1. Zero profits in equilibrium. (Usual argument: if positive profits, the less well off firm can raise wage by a little bit and capture essentially all profits.)
2. Highest wage must be $\bar{w}$. (Else deviate to just below $\bar{w}$. This is profitable by the regularity condition.
3. All must charge $\bar{w}$. (Otherwise high price firm can lower wage.)
4. Indicated strategies are an equilibrium. (Higher wages can't be profitable by the definition of w^*.)