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Joel Sobel

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Imperfect Monitoring

Basic Example: Quantity Competition with Noise

This is a simplified overview. Mailath and Samuelson's text (beginning of Chapter 11) has a higher level treatment. Green and Porter (JET, 1984) introduce a basic model.

1. Two firms.
2. Firms select quantity q_i .
3. Stage game payoff, linear Cournot: $\theta[1 - (q_1 + q_2)]q_i - cq_i$, c is constant marginal cost, θ is demand shock.
4. Firms pick quantities; nature picks θ ; firms observe payoff – strictly speaking market price $\theta[1 - (q_1 + q_2)]$ is observed by both players; θ and opponent's quantity, not public; repeat (with discounting and θ iid).

Analysis

1. Simple static analysis (linear Cournot, with average shock in payoffs).
2. Static equilibrium output larger than joint profit maximizing.
3. Question: If patient players play repeatedly, can they do better than static monopoly?
4. If they can do better, can they do as well as joint profit maximizing?

Routine Computations

Static Duopoly:

$$\theta(1 - 2q_i - q_j) - cq_i = 0 (i \neq j)$$

or

$$q_i = \frac{(\theta - c)}{3\theta} (q_i = 0 \text{ if } c > \theta)$$

$$\pi_i = \left(\frac{\theta - c}{3\theta} \right)^2 \theta$$

Collusion:

$$q_i = \frac{(\theta - c)}{4\theta}$$

$$\pi_i = \left(\frac{\theta - c}{\theta} \right)^2 \frac{\theta}{8}$$

Review

1. Static: Nash equilibrium dominated by collusive behavior.
2. Repetition with patient players can make collusion an equilibrium if θ is known.
3. What if θ is not known?

Some things don't change

1. Can talk about (subgame perfect) equilibrium.
2. Can talk about feasible (expected) stage-game payoffs.
3. Can talk about minmax. (What is it?)
4. Potential to punish exists.

Some things are different

- ▶ In complete information game, you know when someone has deviated.
- ▶ In imperfect monitoring game, you do not.
- ▶ Low profit may be the result of low realization of θ – bad luck – or high output by opponent – cheating.
- ▶ If you punish when profit is low, then you run the risk of punishing someone because of bad luck.
- ▶ If you do not punish when profit is low, opponent will have incentive to cheat.

Hence folk theorem is in doubt.

Approaches

1. Restrict attention to strongly symmetric public strategies: Strategies depend only on public history (that is, player one does not condition on what she produced in past, but only on what is publicly observed). Theoretically this restricts the set of equilibria.
2. Use “optimal penal code approach” and try to characterize equilibrium using essentially two equilibrium profiles: the target profile and the punishment profile. (There is a theory that justifies this approach.)
3. Limit attention to a tractable class of punishments (sometimes not fully compatible with previous step).

Concretely

1. Assume players pick cartel outputs in every history until price falls below a threshold.
2. If price falls below threshold, switch to Cournot (or other profile) for N periods.
3. After N periods, switch back.

Details

1. What is the best N ? (Higher N stronger punishment, which is good before the punishment is triggered and bad afterwards.)
2. What is the best punishment output (maybe static Cournot isn't the best)?

Sample Computation

1. $c = 0$.
2. Want producers to both pick $q_i = .25$.
3. Suppose θ uniform on $[0, 2]$.
4. Suppose punishment starts if $\theta(1 - q_1 - q_2) < T$.
5. Suppose $N = \infty$.

Details

1. Probability of initiating punishment in equilibrium = probability $\theta < 2T$.
2. Equilibrium probability: T .
3. Payoff in equilibrium, V .

$$V = (1 - \delta)/8 + \delta((1 - T)V + T/9)$$

or

$$V = \frac{(1 - \delta)/8 + \delta T/9}{1 - \delta(1 - T)}$$

4. Payoff to deviating to output q :

$$(1 - \delta)q(3/4 - q) + \delta((1 - P)V + P/9)$$

where P is probability of punishment

$$P = \frac{2T}{3 - 4q}$$

How to deter cheating?

Must have $q = 1/4$ solve

$$\max(1 - \delta)q(3/4 - q) + \delta((1 - P)V + P/9)$$

Consequently

$$(1 - \delta)/4 = \delta(V - 1/9).$$

Note: V depends on T . You can check that given δ is sufficiently close to one you can find T to solve equation.

Interpretation

1. It is possible to do better than static Nash average payoffs when players are patient.
2. In constrained family of strategies, discount factor determines the probability of punishment.
3. Punishment is costly (outcomes are not as good as joint monopoly).
4. It is possible to construct “better” equilibria.
5. When do you think that you look for these?

Alternatively

1. Pick four things:
 - 1.1 “target” output levels
 - 1.2 “punishment” output levels
 - 1.3 “target” price to trigger punishment.
 - 1.4 “punishment” price to return to target production.
2. Start by playing target output.
3. Continue doing so as long as price remains high.
4. When price falls below target price, switch to punishment outputs.
5. Continue doing so as long as price remains low.

Incomplete Information

The applications all involve models of incomplete information. We can study these games using the techniques developed using a “trick” of Harsanyi.

Assume that there is an underlying “type” space that summarizes all of the uncertainty.

Player i 's type determines his payoff function and possibly other characteristics.

Assume now that the game is one of imperfect information. It begins with a move of nature – selecting players' types. Nature then tells each player his or her type (but not the types of others). Solve using “standard” techniques.

Formality

Bayesian Game: $\{I, \{S_i\}, \{u_i\}, \{T_i\}, F\}$, $S = \prod_{i=1}^I S_i$, $T = \prod_{i=1}^I T_i$

- ▶ I – player set.
- ▶ S_i – strategy set of player i .
- ▶ $u_i : S \times T \rightarrow \mathbb{R}$ – payoffs
- ▶ T type space (summarizes private information).
- ▶ F distribution on T .

Equilibrium is a profile of $s_i^* : T_i \rightarrow S_i$ such that for each i and $t_i \in T_i$

$$s_i^*(t_i) \text{ solves } \max_{s_i(t_i) \in S_i} \int E u_i(s_i, s_{-i}^*(t_{-i}), t) dF(t | t_i).$$

Note: You need a specification of the strategy for all types (even though only one type is realized). It does make sense for u_i to depend on the entire vector t .